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THERMAL ATMOSPHERIC MODELS

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INTRODUCTION

Makers of atmospheric models could well be intimidated by the wide variety and unusual character of the red-giant stars as revealed by the observations already described, which quite commonly include such features as emission lines, thermal emission from dust shells, maser emission, mass outflow (and inflow), polarization, variability, and rapid evolution. These phenomena present tough questions. To what extent are the atmospheres of M, S, and C stars to be understood simply as cooler examples of ordinary warmer stars? Are inhomogeneous atmospheres common among red giants and supergiants? Will discrepancies due to our present inadequate treatments of convection conceal other theoretical deficiencies? Are chromospheres permanent or transient? By what mechanisms are they produced? Where and how does grain formation and growth occur? What mechanisms are responsible for mass loss, and what are the consequences? With extended atmospheres and temperature inversions, aren't departures from local thermodynamic equilibrium (LTE) likely? Can one neatly separate atmospheres into photospheres, chromospheres, and circumstellar shells? The questions go on and on.

In spite of the uncertainties raised by these questions, atmospheric model makers have historically treated all these more exotic phenomena as secondary and have concentrated first on modeling photospheres. A photospheric model is not only a desirable first step, but it has a rather logical priority since all other structures, processes, and phenomena depend on it. In this chapter, we describe the *static thermal atmosphere* and compare its predictions to observations both to test the validity of the classic assumptions and to distinguish and describe those spectral features with diagnostic value. By thermal atmosphere, we mean an atmosphere based on hydrostatic equilibrium, radiative equilibrium, and LTE. Nonthermal atmospheric models, including nonradiative heating and departures from LTE, are treated in a succeeding chapter (de la Reza, this volume), and more exotic phenomena are described in other chapters.

Several useful reviews of the challenges of computing atmospheres for red-giant stars and using these to interpret observations have already been given. Vardya (1970) gave a broad-brush treatment of the topic from information then available, and Johnson (1972) analyzed the few theoretical models computed to that time. Although advances in modeling atmospheres of red-giant stars have come slowly, it is heartening to note the obvious progress from these early reviews to the present. Remarkably, even the recent excellent general works on stellar atmospheres (for example, Gray, 1976; Mihalas, 1978; Baschek and Scholz, 1982) contain few applications to red-giant stars. More valuable are recent specialized reviews. Carbon (1979) gives a detailed general review of problems and techniques of computation of atmospheres of intermediate and late-type stars from the point of view of the model maker; Gustafsson (1981) writes a topical review of late-type stars from the point of view of the observer; and Johnson (1985) discusses the current availability of models for peculiar red-giant stars. Carbon (1984) examines both the opacity-distribution function and the opacity-sampling treatments of line opacities, the latter by means of a Monte Carlo technique.

PHYSICAL PRINCIPLES

For modeling red-giant atmospheres, workers have adopted the same principles and techniques used in modeling hotter main-sequence stars, for which these principles were perhaps more appropriate. The most obvious exception is in the treatment of line opacity, for which the relatively few lines in hot stars can be treated individually but the millions of lines in cooler stars must be treated collectively. The basic physical principles are hydrostatic equilibrium, radiative equilibrium, and local thermodynamic equilibrium (LTE). The solution of these equations in a plane-parallel horizontally homogeneous geometry constitutes the problem of the *classic stellar atmosphere*.

Hydrostatic equilibrium in a plane-parallel geometry is:

$$dp = -g\varrho dz \quad , \qquad (7\text{-}1)$$

where the total pressure $p = p(\text{gas}) + p(\text{radiation}) + p(\text{turbulence})$, and the surface gravity $g = GM/r^2$ is held constant. Turbulent pressure is often neglected.

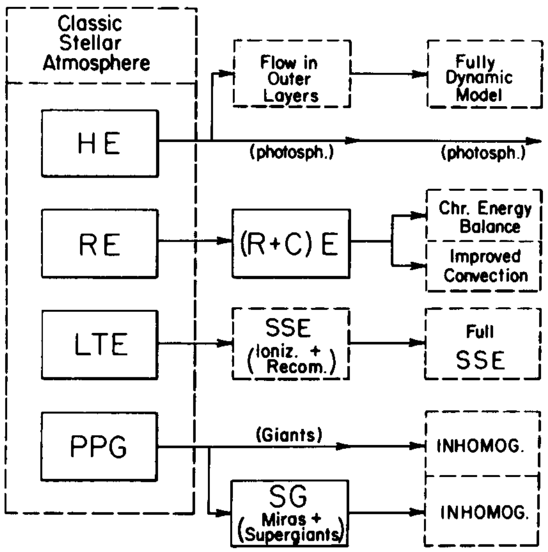
The constancy of the total energy flux is:

$$F = \sigma T_{\text{eff}}^4 \quad , \qquad (7\text{-}2)$$

where F is the energy flux (erg cm^{-2} s^{-1}) and T_{eff} is the effective temperature, defined by Equation (7-2). Strict radiative equilibrium implies $\int F_\nu \, d\nu = F(\text{rad}) = F$, whereas convective energy transport is included through $F(\text{rad}) + F(\text{conv}) = F$. When included, convection has usually been treated by the usual local mixing length (LML) theory without overshoot (Böhm-Vitense, 1958; Henyey et al., 1965; Mihalas, 1978; Lester et al., 1982).

As generally used, LTE includes two key assumptions: (1) concentrations of molecules, atoms, and ions are given by the relevant equations of equilibrium statistical mechanics with the single parameter of electron temperature; and (2) the source function for all bound-bound transitions and for all continuous transitions except for pure scattering is the blackbody radiation (Planck) function, $B(\nu, T)$. Nothing in the modern treatments of bound-bound opacity (opacity distribution functions, opacity sampling (OS), or Voigt-analog/Elsassar-band model; see following section) *requires* this assumption on the line source function; they are all flexible enough to accommodate a simple mixture of pure absorption and pure scattering. (The OS has, however, a clear advantage.) The LTE assumption is made for simplicity and to test this hypothesis in the face of departures anticipated because of the observed departures of the emitted energy flux of cool stars from that of a blackbody and the low gas densities (hence collision rates) in the outer atmosphere.

We attempt to display the present situation and foreseeable developments in Figure 7-1. Principles of the classic stellar atmosphere are shown at the left. Developments or improvements currently available or on the horizon are shown in solid boxes; developments still in the future are shown in dotted boxes. Except for the addition of convective energy transport and the extension to spherical geometry, both of which are, in some form, already in use, generalizations of these classic principles may be slow in appearing. Recall that we are speaking here of stellar photospheres. Some of the improvements labeled "future" are regularly applied in studies of circumstellar (CS) envelopes and mass loss; current progress in these areas



HE = hydrostatic equilibrium
RE = radiative equilibrium
LTE = local thermodynamic equilibrium
PPG = plane-parallel geometry
SG = spherical geometry
CE = convective equilibrium
SSE = steady-state equilibrium

Figure 7-1. Overview of the physical principles underlying current theoretical research on classic thermal atmospheres of red-giant stars. Present physical principles and generalizations are enclosed with solid lines, and possible future generalizations are enclosed with dotted lines.

can be gaged from other chapters in this volume. The rate of future progress can be guessed at by the reader as it has been guessed at by others (Johnson, 1985). With the arrival of information in the ultraviolet, infrared, and radio regions and with the prospects for employing supercomputers to model the entire atmosphere, including mass flow, on the horizon, we appear to be entering an exciting era.

OPACITIES

Continuous Opacities

Many of the opacities for cool stars are similar to, or can be extrapolated from, those at higher temperatures. Throughout most of the photosphere, H^- provides the dominant continuous opacity over most of the spectrum even down to stars of T_{eff} = 2500 K and for a variety of chemical compositions (Johnson, 1982). In the coolest region of the atmosphere, hydrogen may be associated into H_2, and the H^- opacity will become so small that He^- (free-free) will become a dominant continuous opacity. This may be the case in hydrogen-deficient atmospheres as well. Bound-free continua of H I are only of minor importance, and the bound-free continua of He I are completely negligible. (These may be of importance, however, in shocks or in the hot chromospheres of cool stars.) Absorption due to bound-free continua of other neutral elements (Si, Al, Na, Mg, Ca, and Fe) are likewise of minor importance because their absorption edges lie in the ultraviolet, where the stellar flux is weak. Except for H^-, the opacity of negative ions (Vardya, 1970) does not appear to be significant. Although Rayleigh scattering from H I, H_2, and He I—with minor contributions from C I, N I, O I, and N_2—is most significant in the ultraviolet, its effect extends increasingly longward in wavelength as the temperature decreases, and it becomes of major importance in the upper photosphere. Information on cross sections for continuous opacity sources is contained in, for example, Carbon and Gingerich (1969); Kurucz (1970); Tsuji (1971); and Baschek and Scholz (1982); while references may be found in Gustafsson et al. (1975); Mihalas (1978); Kurucz (1979); Johnson et al. (1980); and Dzervitis (1983).

Dust Grains

Wherever condensation occurs in a stellar atmosphere, dust will almost certainly become a major opacity because of its very large absorption coefficient. Many cool stars, especially cool supergiants and Miras, show evidence of circumstellar dust (Lefèvre, this volume). Whether grains commonly form in the outer stellar *photosphere* is still uncertain, despite considerable work on this important problem. Two enormous obstacles to progress in overcoming this uncertainty are the lack of knowledge of both the physical conditions in the outer photospheres (this chapter; Zuckerman; 1980; Goldberg, this volume) and the mechanism of nucleation (Deguchi, 1980; Draine, 1981; McCabe, 1982; Alexander et al., 1983; Gail and Sedlmayr, 1984). Vital to the question of grains in red-giant atmospheres is the location of the condensation point, but this is still quite uncertain (c.f. Draine, 1984). For clean silicate grains in M stars, condensation may occur at a distance (from the stellar *center*) of 3 to 4 stellar radii (Deguchi, 1980) or perhaps closer (Draine, 1981; Alexander et al., 1983). In carbon-rich atmospheres, condensation of graphite may occur at 5 stellar radii (from the stellar *center*), and SiC may condense even closer (McCabe, 1982), perhaps as close as 1.5 stellar radii (Lucy, 1976; Woodrow and Auman, 1982). Several minerals condense at about 1500 K (cf. Draine, 1981; McCabe, 1982; Alexander et al., 1983), and such temperatures might be reached in the outer photospheres of sufficiently cool stars, especially if presently unaccounted for spherical effects (Schmid-Burgk and Scholz, 1981) *lower* the temperature even further than predicted by current models. On the other hand, chromospheric heating may *raise* the temperatures in the outer layers. Clearly, the most favorable site for condensation is the outer atmosphere of a cool Mira variable star (Deguchi, 1980; Alexander et al., 1983), where grain formation may be related not only to mass loss but also to pulsation (Woodrow and Auman, 1982). At present, there is no *compelling* evidence for dust in any cool-star *photosphere* (see, for example, Chapters 3 and 7 and the foregoing references). For this reason, and for the sake of simplicity, dust is neglected in most model atmospheres. If future observations demonstrate the existence of photospheric dust grains, their inclusion will extend atmospheric modeling into a rather different opacity regime.

Spectral Line Blanketing

Before plunging into the complexities of the subject, it is worth reminding ourselves of the central role of the *atomic and molecular data* themselves. Much work lies ahead, particularly for the molecules, and we salute those who pursue this unglamorous but necessary research.

Several excellent reviews of line blanketing are available. A description of LTE line blanketing in terms of physical mechanisms is given by Carbon (1979), and a mathematical analysis is presented by Mihalas (1978). Quantitative non-LTE analyses are provided by Athay and Skumanich (1969), Mihalas and Luebke (1971), and Athay (1972). Much of this development builds on foundations laid earlier by Chandrasekhar (1935), Münch (1946), and Pecker (1951).

From the point of view of the observer, the most striking feature of a stellar spectrum is the removal of light by the spectral lines, an effect referred to as *line blocking*. Since the line represents an increase of opacity and the total flux is required to be constant in radiative equilibrium, the temperature in deeper layers must rise to drive the same flux through the increased opacity, an effect referred to as *backwarming*. In addition, the gas in the outer layers may be heated or, more frequently, cooled. Altogether, the effects are referred to as *line blanketing*.

Physical processes near the surface and their effect on the thermal structure are complex. Purely coherent scattering lines change the temperature very little because they are weakly coupled to the thermal reservoir of the atmosphere (Chandrasekhar, 1935; Münch, 1946; Mihalas, 1978). Purely absorbing lines generally act to *lower* the temperature of the outer layers if they lie on the *redward* side of the flux maximum (e.g., H_2O, CN, and CO); purely absorbing lines lying *blueward* of the flux peak (TiO and most atomic lines) tend to *raise* the temperature of the layers in which they finally become thin. These latter effects can be understood as follows (Krupp et al., 1978; Carbon, 1979; Gustafsson, 1981). Absorption of a red (low-energy) photon by an atom or molecule and the reemission of a (higher energy) photon characteristic of the local temperature in the outer layers results in a net energy loss (i.e., a cooling). In the same way, absorption of a UV photon and reemission of a (lower energy) photon characteristic of the local temperature results in a net energy gain (i.e., a heating). This is a simplified picture of a very complex process, however, and a careful quantitative description, including the effects of departures from LTE and a proper description of the mechanism of line formation may be necessary to understand the physics involved in a particular line (Jefferies, 1968; Athay and Skumanich, 1969; Mihalas and Luebke, 1971; Athay, 1972; Mihalas, 1978). Although line formation in the general case, where LTE does not hold, properly belongs in a chapter on nonthermal phenomena (de la Reza, this volume), we mention it here because of its possible importance in treating line blanketing.

In fact, the effect of a line on the atmospheric structure depends on the line strength relative to the continuum, the variation of the line strength with depth, the wavelength of the line, and the mechanism of line formation. This last consideration is sometimes specified beforehand (e.g., pure absorption or pure scattering), which oversimplifies the actual physics of the problem. Of these effects, the blocking and backwarming depend principally on the line strength, whereas the surface cooling depends principally on the mechanism of line formation, but also on the line strength (Mihalas and Luebke, 1971). Quite generally (cf. Athay and Skumanich, 1969; Athay, 1972), the condition of radiative equilibrium requires $dH/d\tau = 0$, where H is the normalized total (integrated) flux. Although the total flux is conserved, the continuous flux is now no longer conserved but is given by:

$$\frac{dH_c}{d\tau_c} = \frac{M^{-1}}{r_o} ZS_\ell + \int (J_c - J_\nu)\, d\nu = t + c \quad , \tag{7-3}$$

where $M = \sqrt{\pi}\, \Delta\nu_D$ (for a Doppler line); $\Delta\nu_D$ is the Doppler halfwidth; $Z = 1 - J/S_\ell$ is the net radiative bracket; J is the mean intensity averaged over the absorption coefficient; and

$$r_o = \frac{d\tau_c}{d\tau_o} = \frac{d\tau_c}{d\tau_\ell(\nu_o)} \quad . \tag{7-4}$$

Here ℓ and c refer to line and continuous quantities, respectively, and ν designates total (line plus continuous) quantities. The net radiative bracket (Thomas, 1960) or escape coefficient (Athay, 1981), when multiplied by the Einstein spontaneous emission coefficient, gives the *net downward* rate of radiative transitions in a given spectral line. This equation states that the presence of a spectral line causes a net exchange of energy between the line and the continuum—an exchange which may proceed in either direction and may depend sensitively on depth. It is the *atmospheric adjustment* to the requirements that total flux be conserved which produces the changes in the thermal structure referred to as line blanketing.

The c term (Athay, 1972) is proportional to the equivalent width of the line and is defined to be positive if the line is in absorption. This term is then independent of the mechanism of line formation, at least to the first order. If an absorption line is present, it provides a sink for the continuous photons and thus tends to cool the atmosphere. This is always the case in the outer layers, and c therefore acts to cool the outer layers regardless of its position in the spectrum, strength, or depth of formation. As one proceeds into the atmosphere, the line may stay in absorption or may go into emission, depending on the wavelength, the line strength (or value of r_o), and the depth dependence of r_o. Lines in the blue part of the spectrum—blueward of the flux peak, where $J > B$—tend to stay in absorption and therefore cool (provide a sink for continuum photons) at all depths, whereas lines redward of the peak, where $J < B$, tend to go into emission (locally) deep in the atmosphere and therefore produce the considerable backwarming observed. A rapid increase in r_0 deep in the atmosphere increases the tendency of lines to go into emission and therefore increases the backwarming.

The t term (Athay, 1972), which is proportional to Z, the net radiative bracket, tends to zero in the layers deep enough that the line is thermalized. Its principal effect appears near the surface. Although its value depends on the strength of the line and has somewhat different values for lines on the linear, flat, or damping part of the curve of growth, its value is certainly positive, so that it cools. For weak lines, c tends to exceed t and produces the cooling; for medium and strong lines, t exceeds c, often by a large amount, and is responsible for the surface cooling. Often, this cooling of outer layers is accomplished by a few very strong lines such as Na I D, Ca II H and K, Ca I 4227, Mg I b lines, and Mg II h and k lines; in other cases, the more numerous medium strong lines provide most of the cooling except in the extreme outer atmosphere.

What of the lines which *warm* the surface, such as TiO (Mould, 1975; Lengyel-Frey, 1977; Tsuji, 1978a; Krupp et al.; 1978)? Carbon (1979) explains these in a physical way based on analogy with processes in the continuum discussed by Dumont and Heidmann (1973, 1976). In these processes, a major change in opacity occurs near the surface. Such processes can be discussed quantitatively within the above framework (Athay, 1972) if r_0 decreases rapidly with optical depth. Some molecules (TiO, H_2O, and HCN) exist in highest concentration near the outermost layers, where r_0 may change appreciably. In such cases, c becomes negative, and the surface layers are heated. This occurs for both the Schuster-Schwarzschild and Milne-Eddington atmospheres and is independent of the mechanism of line formation. The warming is enhanced by a flatter temperature gradient in the outer atmosphere, a line closer to LTE (ϵ is larger), and a value of r_0 which *decreases* with depth near the surface. An illuminating example of an atmosphere with an opacity discontinuity has also been explored by Mihalas (1969).

In actual calculation of stellar atmospheres, consideration of the detailed processes of line formation is normally avoided through the assumption of pure absorption (or even stronger assumption of LTE) so that the line source is equal to the Planck function for all lines, as shown in Figure 7-1. Occasionally, scattering is included for the strongest lines. Although this assumption may be adequate for deeper layers, its use might lead to fairly serious errors in the upper photosphere. Its only justification is a currently necessary simplicity. In fact, although some progress has been made, a *thorough examination of the mechanism of line formation* and of the possible errors arising from the assumption of pure absorption *is still lacking for even the strongest lines in the red-giant stars* (Carbon, 1979; Gustafsson, 1981; de la Reza, this volume). Research on this problem can be expected to yield significant new knowledge.

Treatments of Line Opacity

Even with the assumption of pure absorption in the line-source function, accounting for the rapid, almost random variation of the bound-bound absorption coefficient with wavelength across the entire spectrum, as implicit in the carbon-star spectrum shown in Figure 7-2 (Querci and Querci, 1975b), is a formidable task, and some statistical procedure becomes necessary. The successive steps in treating such opacities form the backbone of a history of cool-star atmospheric modeling and the successful treatment of at least certain atomic and molecular opacities represents one of the significant recent triumphs in stellar atmospheres.

Intuitively, one knows that an increase of opacity in a given frequency band will inhibit the flow of radiation there, and flux constancy then requires an increase in radiative flux at neighboring frequencies. This leads naturally to the idea of a harmonic mean (HM) opacity (i.e., K^{-1} is the integral of the reciprocal of the

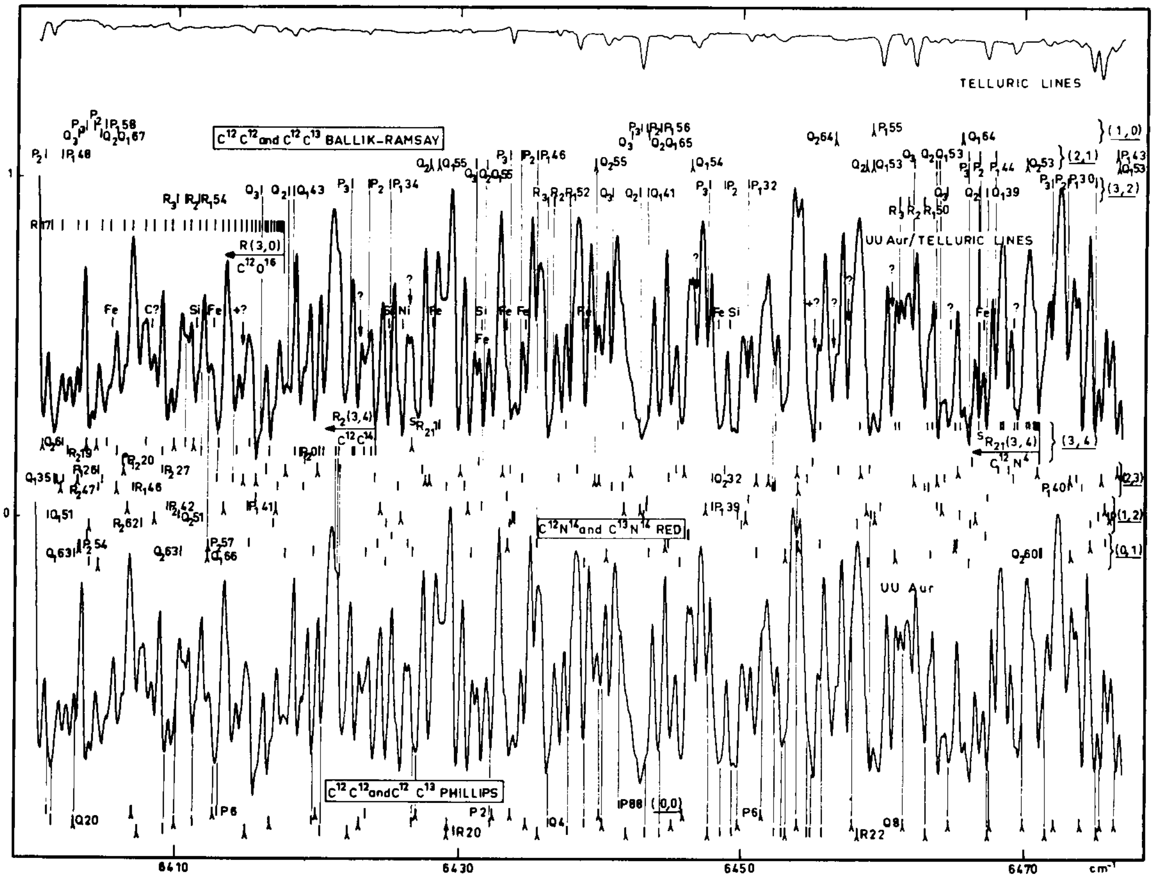


Figure 7-2: The observed spectrum of the N-type carbon star UU Aur in the region 6400 to 6480 cm^{-1} (bottom graph) and the same spectrum corrected for weak telluric lines (middle graph). Many lines of C_2 and CN are identified (from Querci and Querci, 1975b).

total absorption coefficient over a certain spectral interval divided by that interval). Unfortunately, different sources are not easily additive in an HM opacity, which makes its use cumbersome at best. Furthermore, the HM underestimates the opacity (Querci et al., 1972; Carbon, 1974, 1979; see also Figure 7-4). It has been used in only one set of models (Auman, 1969), in which H_2O was the only opacity treated.

A markedly different approach is represented by the straight mean (SM) opacity, which is simply the average integrated absorption coefficient of all lines in a spectral interval. An SM absorption coefficient (per molecule) for each species in each interval is fitted by a simple polynomial as a function of temperature, and the fitting coefficients are tabulated once for all (Alexander and Johnson, 1972; Johnson, 1974). Absorption coefficients for several absorbers are easily combined, and models are quickly calculated. The weakness of the SM is that the averaging process spreads the opacity over an entire spectral region, filling in the narrow spectral windows through which much of the radiation in reality escapes. The SM therefore overblankets the model, as has been elucidated by several studies (Querci et al., 1972; Johnson, 1974; Carbon, 1974, 1979). The accuracy of the SM obviously improves as the spectral interval over which the smoothing is done is decreased. Johnson et al. (1972) show an example for CN. For comparable spectral intervals, the SM can be thought of as a one-picket opacity distribution function (see also Figure 7-4).

The opacity distribution function (ODF) was described by Strom and Kurucz (1966), and its modern use dates from about that time. In a given spectral interval (of, say, 50 Å), one rearranges the absorption coefficient in wavelength to obtain a monotonic ordering, the result having the appearance of one giant spectral line (Figure 7-3). This new monotonic function—termed the opacity distribution function—is then approximated by a series of pickets as shown. (Care must be taken, of course, to flip the giant line from side to side in alternate intervals and depths to avoid systematic effects (Gustafsson et al., 1975).) Various mathematical descriptions of the ODF are available (Querci et al., 1971; Carbon, 1973, 1979, 1984; Kurucz et al., 1974; Mihalas, 1978), and several applications have been made (Querci et al., 1974; Querci and Querci, 1975a; Gustafsson et al., 1975; Kurucz, 1979; Eriksson et al., 1985).

A representative ODF of CN and C_2 for conditions typical of carbon stars is shown in Figure 7-4 (Querci et al., 1971). Although the absorption of C_2 is much less than that of CN, it influences the ODF significantly. Also shown are the (constant) values for the SM and HM opacity; clearly the SM overestimates, and the HM underestimates, the true opacity. Similar comparison of SM, HM, and ODF opacities for CN in a carbon-rich model atmosphere was made by Carbon (1974).

Calculating the ODF is tedious, but construction of models proceeds rapidly when the ODF is available. If, however, the ODF is sensitive in a given wavelength region to several opacity sources, the relative strengths of which might depend on a parameter (such as chemical composition) that is not known beforehand, one could be forced to a time-consuming iterative procedure (cf. Carbon, 1979, 1984). At least some of this liability could be overcome by a two-step procedure in which ODF's are formed separately for each species and are then

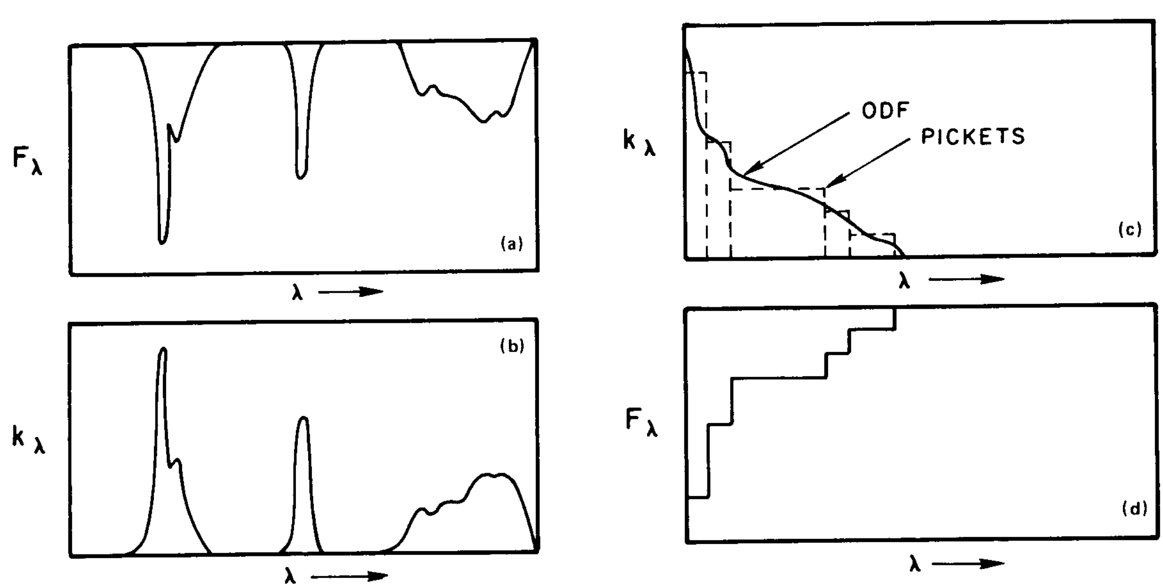


Figure 7-3. (a) Hypothetical spectrum over a spectral interval between λ_1 and λ_2. (b) Hypothetical line absorption coefficient over this interval. (c) The information in Figure 7-2b rearranged by strength of absorption to form an ODF or giant line (solid line) as described in the text. A possible six-picket representation of the giant line is shown by the dotted line. (d) The flux emitted by the picket representation of the opacities in (c).

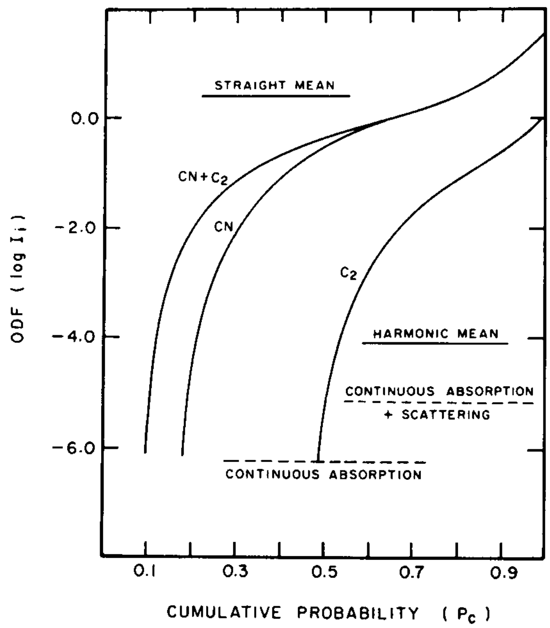


Figure 7-4. Comparison of various opacity treatments for CN and C_2 in the wave-number interval 6320 to 6420 cm^{-1}. The graph shown (from Querci et al., 1972) is for $T = 2500$ K, $P = 1$ dyne/cm^2, and microturbulent velocity VT = 5 km/s.

reshuffled to account for the changing depth dependence of the opacities (Tsuji, 1976a). Recently, in what appears to be a significant advance, the Swedish group has demonstrated the practicability of this suggestion by actual calculation and has further shown how the accuracy of the ODF can be increased by spline-fitting (Olander, 1981; Saxner and Gustafsson, 1984). This method allows the ODF to accommodate even the well-known problems of the depth-dependent combination of CN, C_2, and CO opacities in cool carbon stars.

Another drawback to the ODF arises from the rearranging of the absorption coefficient and its representation by a series of pickets; in this smoothing, all information regarding individual absorbers is lost. To ascertain the effect of a change in even one spectral line, as might arise from departures from LTE in even one atomic species, it is necessary not simply to recompute an atmosphere with this changed value, but to recalculate the ODF. It is difficult even in principle to determine how this weakness of the ODF method can be overcome (Carbon, 1984). For certain general problems, in which self-consistency between input parameters and results is necessary, the ODF may be too restrictive. For problems in which all parameters to which the absorption is sensitive can be specified beforehand, however, the ODF method is convenient and accurate. In addition, the ODF appears to be capable of describing the opacity in a differentially moving atmosphere (cf. Mihalas et al., 1976).

Suppose one begins to calculate model atmospheres without averaging the opacity in any way (i.e., suppose one were not constrained by the amount of computing time available). Under these ideal circumstances, a straightforward procedure would be to calculate the integrated flux and its depth derivatives by a direct integration over *many* monochromatic fluxes. Even though the variation of the absorption coefficient with frequency is rapidly changing, its integral can be obtained to any desired precision by taking a sufficient number of frequency points. This approach is called opacity sampling (OS), for there is no attempt to use all the opacity information; rather, one samples until one has sufficient.

How many frequency points are needed? Very favorable answers are already available; even for complex opacities, only 500 to 1000 frequency points are necessary if they are scattered over the entire spectrum and are randomly chosen with regard to opacity features (Peytremann, 1974; Johnson and Krupp, 1976). Models computed with even as few as 100 frequency points match fairly well a more exact model in the photosphere, but depart from it in the outer layers, and several models with different sets of 100 frequencies lie within a certain envelope in a T–τ diagram. As the number of frequency points is increased, the envelope narrows, and the envelope of models with 500 points has a spread which ranges from near zero deep in the photosphere to ± 50 K near the surface (Johnson and Krupp, 1976). The final model is thus "converged" in the sense that, for more than about 500 points, *every set of points gives the same model*. Related interesting numerical experiments with an ODF and an OS calculation are reported by Gustafsson et al. (1975) and Gustafsson (private communication, 1984). Such characteristics put the OS method easily within the capabilities of current computers.

The power of the OS method arises from its simplicity and its retention of all information on all lines which influence the transfer of radiation at the selected frequencies. Everything is under the control of the investigator. New values of such factors as chemical composition, turbulent velocity, or even departures from LTE are easily accommodated. This flexibility comes at a cost, however, because the construction of each atmosphere demands much computing. Normally, the absorption of several tens of thousands of spectral lines (selected from a much larger sample) are accounted for in each calculation, making each iteration fairly time-consuming. Although some tricks (such as beginning with a small frequency set and using a larger set only for the final iterations) can save considerable computing, the OS method generally requires more machine time per model than other methods. Furthermore, even after a model is converged, the computation of the emergent flux requires the calculation of fluxes at many frequencies within each element of spectral resolution, and the fluxes are then smoothed to the desired resolution. Finally, one must be cautious of OS models in the extreme outer photosphere, in which the few strongest spectral lines which control the energy balance and the thermal structure may not be properly sampled. Quantitative investigation of this point is needed. Use of the OS method for radiative transfer in an atmosphere with a velocity gradient is also very awkward (Carbon, 1984) because of the shifting of line centers with depth.

Although the ODF and OS may appear to be competitors, they are in some sense complementary (cf. Carbon, 1984). When all parameters can be specified beforehand (e.g., in constructing a grid of stars of known composition), the ODF method is preferable because of its speed. When absorption-affecting parameters are to be varied (e.g., in constructing a sequence of stars differing in C/O or in accounting for departures from LTE), the OS is superior. (Additional insight can be gained from reading Gustafsson et al., 1975; Johnson and Krupp, 1976; Carbon, 1979; Gustafsson, 1981; and especially Carbon, 1984; and Saxner and Gustafsson, 1984). *We suspect that both the ODF and OS are capable of further development and look forward to these improvements.*

Less complicated than implied by the name (Voigt-analog/Elsasser-band model, or mercifully VAEBM), an offspring of the ODF has been developed (Tsuji, 1976a) and used with success in constructing red-giant atmospheres. The method proceeds from a generalization (Golden, 1969) to Voigt profiles of the Elsasser band model, which treats the opacity of a complex molecule by a picket-fence model consisting of an infinite set of equally spaced spectral lines of equal strength. Although real spectral lines have different strengths, shapes, and widths, one can nevertheless define average values of these quantities, adding pickets as needed. Even overlapping lines can be roughly accounted for. Restrictive as these assumptions appear, the VAEBM representation is, over a spectral interval sufficiently small that the Planck function does not vary appreciably, formally equivalent to the ODF described earlier (Tsuji, 1976a), and it has been exploited with good results. However, one can easily imagine the method failing in the case of overlapping opacities, especially with different depth dependences (Saxner and Gustafsson, 1984). In short, the VAEBM is a simple ODF, with the strengths and weaknesses of the ODF in addition to the more severe approximations by which the VAEBM gains its simplicity. On the other hand, it has the advantages of speed and the power to treat, at least crudely, even such complicated molecules as H_2O. Certain molecular VAEBM opacities have been compared to Rosseland (harmonic mean) opacities and JOLA opacities (just overlapping line approximation) in an interesting paper (Tsuji, 1971).

Like a phoenix, SM opacities never really die, and they have recently been in the news again because of their use in spherical models. A comparison of Rosseland opacities from SM and OS treatments (Scholz and Tsuji, 1984) finds good agreement for solar composition, but for a carbon-rich composition, the OS opacities are much smaller. Unfortunately, the agreement for solar composition is largely fortuitous because the OS treatment included H_2O as SM opacity (Johnson et al., 1980), and at the temperature used (T = 3000 K), the OS opacities are in fact dominated by H_2O. Accord between "OS" and "SM" is therefore simply accord between the SM of Johnson and the SM of Tsuji. Clearly, the Rosseland opacity must always be greater for SM than for OS or ODF, because the SM approximation fills in all opacity windows and acts, in a given spectral interval, exactly as an additional *continuous* opacity source.

In the following thought experiment, we imagine a continuous opacity and a superposed line opacity. Let the lines (rectangular for simplicity) cover half the spectrum and have an (equal) absorption coefficient 100 times that of the continuum. If calculated by the ODF or OS method, the Rosseland opacity is approximately *twice* that of the continuum alone since that half of the interval covered by lines is extremely opaque. If calculated by the SM, the average line opacity would be *50* times the continuous opacity, and the Rosseland opacity would also be increased by a factor of *50*. Clearly, *Rosseland opacities and SM opacities are not to be mixed.* Yet consider the absorption of a molecule with enormous numbers of weak lines, such as H_2O or HCN. If the 100000 strongest lines of HCN make little difference in an atmosphere (Tsuji, 1984), but 6 million weak lines make a large difference (Jørgensen, 1985), we understand the reason. Such an enormous number of lines fills in *all* the spectral windows, and like the SM, acts at that depth as an additional *continuous* opacity. If a molecule contributes a haze of lines (even weak ones), it may greatly increase the opacity, and such polyatomic opacities (Auman, 1969; Tsuji, 1984; Eriksson et al., 1984) must be included (laboriously) in at least the coolest red-giant atmospheres. A similarly large Rosseland opacity can be obtained by filling in the opacity windows with isotopic lines and very large microturbulant broadening (Sharp, private communication, 1984).

THEORETICAL PHOTOSPHERIC MODELS

As might be expected, the passage of time has seen a steady improvement in theoretical photospheric models, particularly in regard to the treatment of opacity. It is convenient, then, to separate models into two groups according to the opacity approximation employed: (1) first-approximation opacities (SM, HM, or similar treatment), and (2) second-approximation opacities (ODF, OS, or VAEBM treatments). *We include in this chapter only giant and supergiant models with effective temperatures below 4000 K.*

First-approximation models have already been summarized and extensively discussed (Vardya, 1970; Johnson, 1972, 1974; Kipper, 1973; Carbon, 1979; Gustafsson, 1981). Although they are now generally superseded by superior models with the same range of parameters, these earlier models were the basis of several noteworthy discoveries. Perhaps the most significant was that, contrary to common belief, convection (in the usual mixing-length formulation) was of minor importance in the photosphere itself—both for M stars (Auman, 1969) and for carbon stars (Johnson, 1972). Somewhat surprisingly, even though models with SM and HM opacities have quite different structures, computed molecular column densities agree very well (Goon and Auman, 1970; Johnson et al., 1975), indicating that these latter are rather insensitive to the details of the atmospheric structure. The large scale heights observed in stars such as ζ Aur were duplicated by theoretical models with appropriately low values of surface gravity (Johnson, 1972). The observed flux from a cool carbon star was fairly well represented by the predicted flux from an appropriate model (Johnson, 1972), except that the model flux was too high in the blue and the CN bands were too deep. Older atmospheric models were used for several projects: (1) to examine the remarkable cooling by CO in the outer layers of stars (Johnson, 1973); (2) to interpret the infrared spectra of several carbon Mira stars in support of the hypothesis of C enrichment rather than CNO processing (Thompson, 1977); (3) to study the atmosphere of Betelgeuse (Fäy and Johnson, 1973); (4) to deduce, from IR observations, approximate effective temperatures of K and M giants and supergiants (Scargle and Strecker, 1979); and (5) to infer the ratio $^{12}C/^{13}C$ in five carbon stars (Olson and Richer, 1979). Finally, to explore the gross molecular features of S stars, Piccirillo (1980) computed 85 SM model atmospheres for $2500 \leqq T_{eff} \leqq 3500$ K and log $g = 0$ for compositions enriched in C to give ratios of C/O in the range 0.6 (solar) to 1.00. A point of interest here is his finding that the thermal structures of some of these SM models were similar to OS models. Several conclusions regarding C/O and s-process enhancements in S stars emerged from a comparison of his molecular column densities and observed band strengths—perhaps the most significant being that, in certain S stars, C/O may be nearer 0.9 than 1.0.

In considering models with *second-generation opacities*, we continue to distinguish models which are oxygen-rich (M stars) and carbon-rich (C-stars). S stars, which are intermediate, will be discussed along with C stars.

Models for M Stars

An overview of currently available models for solar composition is given in Figure 7-5 (from Johnson, 1985), where models are shown in the parameter space of effective temperature and surface gravity (log g). All published models with the assumptions of Figure 7-1 and plane-parallel geometry (PPG) and based on ODF, VAEBM, or OS opacities are shown; they are identified as BEGN (Gustafsson et al., 1975; Bell et al., 1976a). T (Tsuji, 1976a;

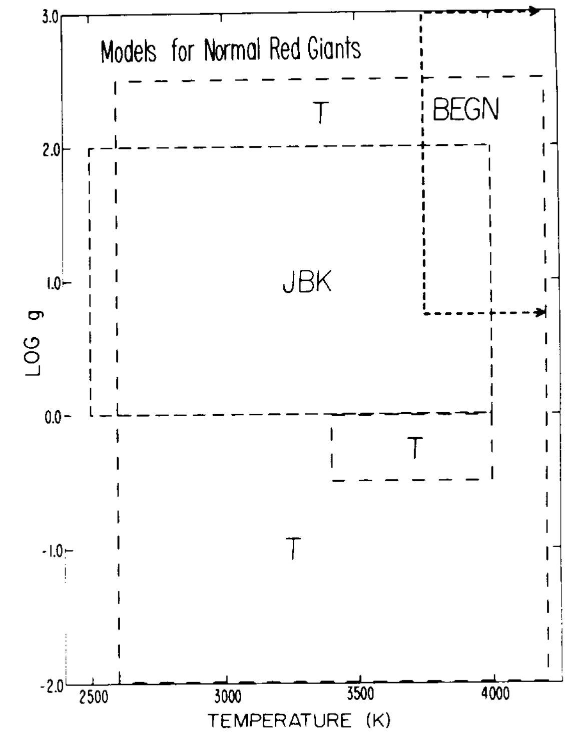


Figure 7-5. Overview of available models for normal red giants of solar composition displayed in the space defined by the surface gravity (log g) and the effective temperature. References are given in the text (from Johnson, 1985).

1978a), or JBK (Johnson et al., 1980). In each group, certain additional models with variations in parameters other than those shown as coordinates were calculated. Models of Tsuji (1976a, 1978a) are purely radiative, whereas models of Gustafsson et al. (1975), Bell et al. (1976a), and Johnson et al. (1980) also include convection. Although it may appear that this parameter space is fairly well covered by models, the coverage is sparse in places, and several principles of the classic atmosphere may be stretched rather badly toward lower temperatures. Perhaps the worst assumption is the crude treatment of H_2O opacity, but the assumption of PPG and even homogeneity may not be met in the coolest, most extended stars.

Designed to represent Population I and II giants, the grid of Gustafsson et al. (1975) and

Bell et al. (1976a) spans the ranges 3750 K $\leqq T_{eff} \leqq$ 6000, 0.75 $\leqq \log g \leqq$ 3.0, and -3.0 $\leqq$ [A/H] $\leqq$ 0.0. Altogether, 103 models based on ODF opacities for CO, CN, OH, NH, MgH, and atomic lines were computed. No TiO lines were included, which unfortunately compromises the accuracy of models with $T_{eff} \leqq$ 4000 K—precisely those of interest here. Convection was treated by the mixing-length theory with a mixing length 1.5 times the pressure scale height. Although turbulent pressure was neglected, a study of the possible effects of its neglect was made. For a model from near the middle of their grid, Gustafsson et al. (1975) find that, for v(turb) = 5 km/s, P(turb) becomes an important component of P(total), but the thermal structure [$T(\tau)$] is affected only slightly except in the convection zone. The surface cooling of CO lines was confirmed quantitatively, and CN lines were shown always to backwarm the entire atmosphere. The authors conclude (Gustafsson et al., 1975) that self-consistent LTE models with as many as 10^5 wavelength points are possible, but a much smaller number of points is satisfactory for either the ODF or OS method. The temperature uncertainty due to all accumulated errors is estimated at 100 K in the outer layers, 20 to 30 K through most of the photosphere, and 100 K in the convective zone. Uncertainties due to the assumptions of LTE and homogeneity, which were not investigated, could be considerably larger. Several applications of these models to astrophysical questions are described in the section *Spectral Distribution of Energy Fluxes* (see also Gustafsson, 1981).

To find a middle ground between the crude approximations inherent in SM opacities and the tedious computations required for the usual ODF opacities, Tsuji (1976a) used the VAEBM method described earlier to treat the opacities of OH, CN, CH, TiO, MgH, SiH, CaH, CO, and H_2O (some approximated more carefully than others) and construct 11 models for supergiant stars. They range in effective temperature from 3400 to 4000 K and log g = 0.0 or -0.5. Turbulent pressure is included in the hydrostatic equilibrium equation with velocities of 3, 6, and 12 km/s, but convection is neglected. In a companion paper, Tsuji (1976b) uses these models for a specific discussion of Betelgeuse, for which he deduces a value of T_{eff} = 3900 ± 150 K.

A grid of 22 models for K and M giants based on these VAEBM opacities was later computed and used to define a new relation between spectral type and effective temperature (Tsuji, 1978a). Based on LTE, hydrostatic equilibrium (including radiative and turbulent pressure with a turbulent velocity of 3 km/s), and radiative equilibrium (convection was neglected), these models were computed for the ranges 2600 $\leqq T_{eff} \leqq$ 4200 K and -2.0 $\leqq \log g \leqq$ 2.5. Predicted fluxes were then compared to photometry of a number of cool stars to fix the relation between effective temperature (from the models) and spectral class (from the stars). Tsuji emphasizes the desirability of comparing infrared fluxes for finding best fits because of the relative insensitivity of the infrared flux to opacity variations. Yet we wonder whether a fit also to the visual region (with its much steeper slope) as well might fix the temperature of a star with greater precision. Many of these models predict substantially too much flux in the visual region (a common problem).

Based on the ATLAS computer program (Kurucz, 1970), Johnson et al. (1980) calculate 40 atmospheres with solar composition and the parameters: T_{eff} = 2500, 2750, 3000, 3200, 3400, 3600, 3800, and 4000 K and log g = 0.0, 0.5, 1.0, 1.5, and 2.0. Convection is included, but turbulent pressure is neglected. Atomic lines and molecular lines from CN, CO, C_2, TiO, OH, NH, CH, and MgH are treated by the OS method; H_2O is added as an SM opacity. In a sense, these results complement those of Gustafsson et al. (1975) to provide a complete set of models for cool giants and supergiants for all temperatures. Although this work is the most complete now available for very cool stars, the weakness of several assumptions should caution the reader against uncritical acceptance of every value. Treating H_2O as an SM opacity overblankets the coolest models

($T_{eff} \leq 3200$ K) (Auman, 1969; Tsuji, 1971), and this is probably the most serious defect. Neglect of grain formation in the coolest models may also be incorrect. The temperature and density in the surface layers of the supergiant ($\log g = 0.0$) models might be even lower if sphericity were included.

How well do the models agree where they overlap? A comparison of $T(\tau)$ has been made, for the particular case of (4200/2.25/solar), between a Bell et al. (1976a) (ODF) model and a Johnson et al. (1980) model (Johnson and Krupp, 1976). All opacity parameters were identical, but the opacity treatment, as well as the computer codes, was different. The excellent accord in $T(\tau)$ — within ± 30 K except in the extreme outer layers — indicates that both the ODF and OS methods are satisfactory. In the extreme outermost layers, the temperature is fixed by those few strong lines which still have some opacity (i.e., for which $\tau(\nu) \approx 1$), and these may not be accurately treated by the OS method (Carbon, 1979; Johnson et al., 1980). Departures from LTE and possible chromospheric heating are also more likely in these superficial regions. Since TiO opacity, which is important in warming the outer layers, is neglected by Gustafsson et al. (1975) and Bell et al. (1976a), their coolest models ($T_{eff} < 4000$ K) are not recommended (McGregor, 1980).

A parallel comparison of a model (4000/1.5/S) with VAEBM opacities from Tsuji's grid with a similar model of Bell et al. (1976a) shows that the two agree within ± 50 over most of the atmosphere, but the former is hotter in the outer layers and cooler in the deeper layers (by up to 130 K). Emergent fluxes from the two also agree well, whereas similar fluxes from an SM model display molecular bands which are too deep (Tsuji, 1976a).

The atmospheres of Johnson et al. (1980) form a sequence parallel to that of Tsuji (1978a), and together these constitute the only models for M giant stars with realistic opacities. A comparison of the T-τ relation between models from each set has been made by Johnson et al. for the parameters 3600/0.0/S, and the agreement is excellent. This is the more impressive because the computer codes and molecular data were independent and the methods of treating the line opacities were different (VAEBM by Tsuji and OS by Johnson et al.).

Models for Carbon and S Stars

An overview of the available models for carbon and S stars (i.e., with C/O > 0.6) is provided by Figure 7-6 (Johnson, 1985), which shows the regions in the parameter space of effective temperature and the ratio of carbon to oxygen covered by various sets of models. Figure 7-6 is, of course, a two-dimensional projection of the multidimensional space whose axes are the parameters specifying these models. Be-

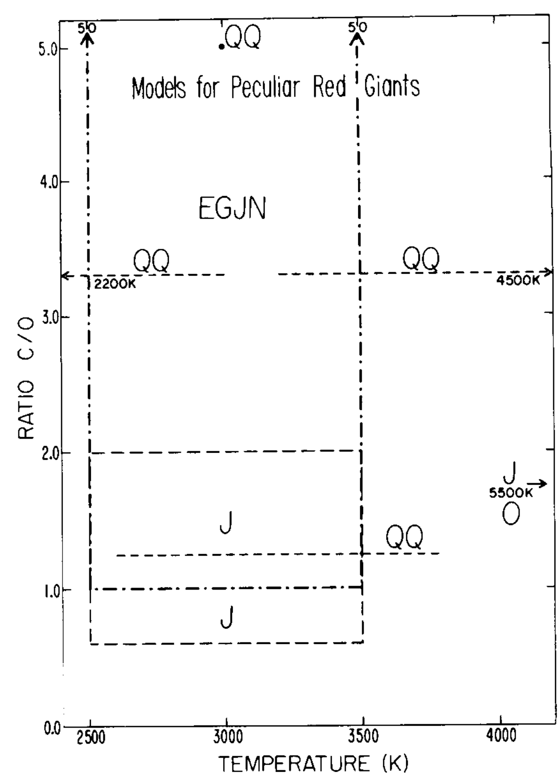


Figure 7-6. Overview of available models for peculiar red giants (M, S, R, and N-type stars) displayed in the space defined by the carbon-to-oxygen ratio and the effective temperature. References are given in the text (from Johnson, 1985).

sides surface gravity, they include the abundance of *all* significant elements, especially C, N, and O. Clearly, Figure 7-6 is a considerable simplification, and sets of models shown as lines (such as QQ) contain models varying in some parameter other than the two used as coordinates—in this case, N/H and log g.

Based on an ODF which included lines of the molecules CN, C_2, and CO, but no atomic lines, the French group (Querci et al., 1974) constructed 12 carbon-star model atmospheres with effective temperatures of 4500, 4200, 3800, and 3400 K and surface gravities of $g = 0.1$, 1.0, 10 cm/s^2. These were later supplemented with 9 cooler models having effective temperatures of 3000, 2600, and 2200 K (Querci and Querci, 1974). Convection was neglected, but a turbulent pressure $P(\text{turb}) = \beta \varrho v^2$ was used in the hydrostatic equation. Values of $\beta = 1$ and $\beta = 0$ and $v = 5$ km/s were chosen to agree with the microturbulent velocity deduced earlier from the spectrum of UU Aur. Solar abundances were used for all elements except CNO, which were fixed at H/C/N/O = $1/4.1 \times 10^{-5}/1.48 \times 10^{-3}/1.25 \times 10^{-5}$ (C/O = 3.2 and N/C = 37), to represent a composition produced by CNO processing and mixing. The effect of molecule line blanketing is clearly seen in their models (Figure 7-7); for successively lower temperatures, the molecular line blanketing increasingly cools the surface layers and warms the photosphere compared to the unblanketed models. These authors found: (1) considerable surface cooling and photospheric heating due to CO, C_2, and CN; (2) good agreement in thermal structure (perhaps surprising) with certain SM models; (3) fair agreement—slightly better than with a blackbody—of predicted fluxes, except in the visual region, with wideband photometry of TX Psc for models with $T_{\text{eff}} = 3000$ and 3400 K. An additional set of 14 models with compositions representing both CNO cycling and triple-alpha processing were calculated, and certain molecular features characterizing each composition were noted (Querci and Querci, 1975a). Synthetic spectra from these models are discussed later.

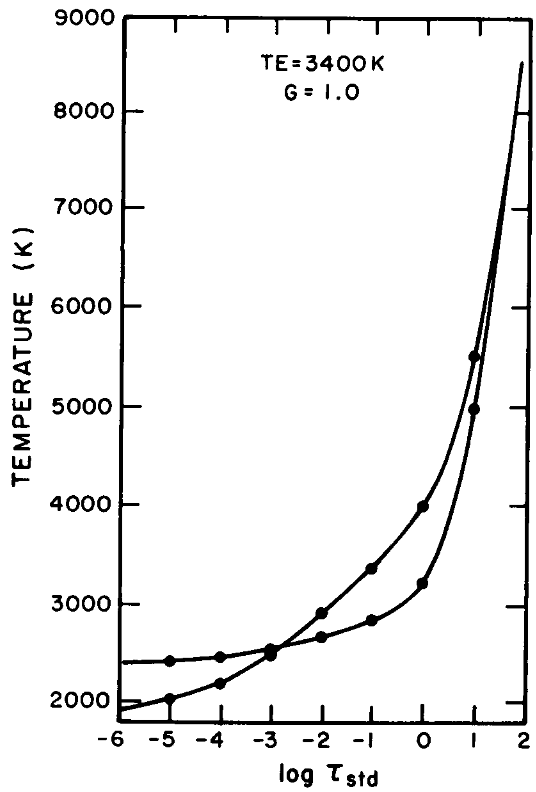


Figure 7-7. Effect of molecular line opacity. At an effective temperature of 3400 K and surface gravity of 1.0, a photospheric model for a carbon star with C_2, CN, and CO opacity is compared to an identical model without these (Querci and Querci, 1974). In each case, the blanketed model is cooler at the surface and warmer in the photosphere.

Another set of 27 models for M, S, and C stars has been produced with OS opacities for atomic lines and the molecules CH, NH, OH, MgH, CO, CN, and C_2 (Johnson, 1982). Three series of models were computed, all with log $g = 0.0$, at effective temperatures of 2500, 3000, and 3500 K. Beginning with a model of solar composition, carbon is added incrementally to give C/O ratios of 0.60 (approximately solar), 0.90, 0.95, 0.98, 1.00, 1.02, 1.05, 1.20, and 2.00. Convection is included in the usual mixing length formulation with $\ell/H = 1.0$; turbulent pressure is neglected. The effect of C/O on the structure is illustrated in Figure 7-8, which displays models with C/O = 0.60, 1.00, and 2.00—possibly representing a cool M supergiant, an S star, and an N-type carbon star, respectively. For comparison, a model with solar composition but *no* line blanketing (labeled 0.60*) has the expected form: a steep gradient in the photosphere and a flat outer portion with a boundary temperature T_0 = 2300 K. An identical model that includes atomic and molecular opacities is warmer throughout the photosphere due to the backwarming and cooler (due mainly to CO and H_2O) in the outer layers (outside the graph shown) with T_0 = 2100 K. As carbon is added and free oxygen is reduced by the formation of CO, the partial pressures and opacities of H_2O and TiO decrease. Recall that H_2O cools near the surface and backwarms the photosphere, while TiO warms the surface layers (Krupp et al., 1978; Gustafsson and Olander, 1979; Gustafsson, 1981). This decrease in backwarming as H_2O and TiO decrease in abundance causes the temperature drop noted. In the outer atmosphere, the increase in CO cooling as carbon is added more than compensates for the loss of H_2O cooling, and the outer layers continue to cool (Carbon,

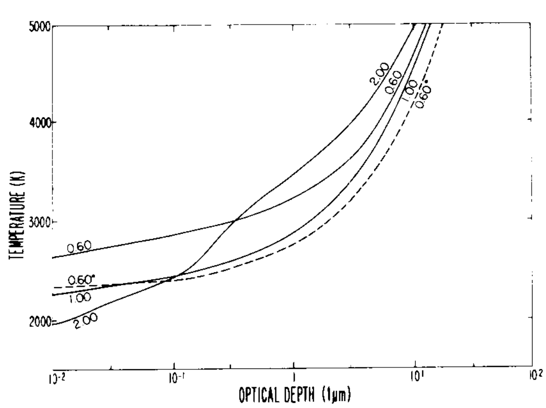


Figure 7-8. Effect of the enhancement of C on the thermal structure ($T(\tau_1)$) of a model with (3000/0.0/solar except C) (from Johnson, 1982). Optical depth is measured in the continuum at 1 μm. The parameter is C/O. A dotted line is a model with solar composition (C/O = 0.60) but without any line blanketing.

1974; Heasley et al., 1978; Carbon, 1979). The remarkable resemblance between the model with C/O = 1.0 and the unblanketed model illustrates well that an S star has little molecular line opacity.

As additional carbon is added beyond C/O = 1.0, the temperature increases again throughout the photosphere due to the backwarming of CN and C_2 (Querci et al., 1972, 1974; Querci and Querci, 1975a; Sneden et al., 1976). In the outer layers, the temperature seems to decrease. In actuality, the *boundary* temperature (1400 K) does not fall, but it is reached at a larger value of optical depth as C/O increases; that is, this cool outer layer deepens as carbon is enhanced (Johnson, 1982). Figure 7-8 illustrates that, because an enhancement of carbon changes the atmospheric structure, spectral features—including those of elements other than carbon or its compounds—may also change. This point has previously been made from a study of molecular column densities (Johnson et al., 1975; Piccirillo, 1980; Johnson and Sauval, 1982), approximate band strengths (Scalo, 1973), and synthetic spectra (Querci and Querci, 1976).

Of great interest is the value of the boundary temperature and the thermal structure of the outer layers of red-giant atmospheres. In his review of model atmospheres for intermediate and late-type stars, Carbon (1979) pointed out that, in the discovery paper (Johnson, 1973), the cooling due to CO (based on SM models) was twice that found in later studies with more accurate ODF opacities (Gustafsson et al., 1975). In this present case, the cooling of the M supergiant is about 200 K, whereas that of the S star and carbon star are about 900 K. In addition, Johnson et al. (1980) list three considerations which might further *lower* the boundary temperature: (1) models may not be calculated at sufficiently small optical depths to reach the true value of T_0; (2) the OS method may fail to sample the strongest lines in the outermost layers and therefore underestimate the cooling; and (3) neglected sphericity effects may lower T_0. A cooler surface layer will increase the chance for grain formation in the photosphere (Lucy, 1976; Menietti and Fix, 1978; Deguchi, 1980; Schmid-Burgk and Scholz, 1981; Draine, 1981; Alexander et al., 1983). On the other hand, chromospheric heating may *raise* the boundary temperature. At present, the boundary temperatures for these stars are unknown, but the value of T_{min} may be obtainable from high-resolution International Ultraviolet Explorer (IUE) observations, and such investigations are strongly encouraged.

Although the R stars, at least those of spectral class R0 to R3, correspond more closely to K than M stars, they are included in this volume because of their carbon-rich spectra and advanced evolutionary state. Models have been computed by two groups. The Swedish group published 20 models (Olander, 1981) with effective temperatures in the range 3800 to 4800 K. An interesting aspect of this project is the attempt to circumvent some of the drawbacks of the ODF (Carbon, 1979) by reshuffling the ODF's of different species. The author concludes that: (1) because of the relatively high temperatures of R stars, changes in compositions do not drastically affect the temperature structure; (2) raising the N abundance causes a slight backwarming ($\leqq$ 100 K) and surface cooling due to the increased CN opacity; (3) the effect of changes in the O abundance is very slight; and (4) raising the microturbulent velocity from 2 to 5 km/s causes a slight warming throughout the atmosphere, as was found for G and K giants by Gustafsson et al. (1975) and Bell et al. (1976a) and for cooler carbon stars by Johnson (1982). At approximately the same time, the Indiana group produced 11 models for R stars (Johnson and Yorka, 1985). These span a slightly higher temperature range (4200 to 5600 K), and surface gravity is fixed at either log g = 2.0 or 3.0. The log abundances of H/C/N/O were taken as 12.00/9.09/8.31/8.55, to give C/O = 1.74, although models were calculated for the additional values 0.60, 1.00, and 2.50 at a temperature of 4600 K. Yorka (1981) found the two sets of models to be in excellent agreement, and Dominy (1984) used them to help infer the effective temperatures of a set of warm R stars (R0 to R3).

None of the carbon-star models published to date contains polyatomic opacities (except for a few SM models by Johnson, 1974), but the importance of HCN in cool carbon stars has recently been reemphasized (Eriksson et al., 1984; Tsuji, 1984; Jørgensen, 1984). A large set of models for $2500 \leqq T_{eff} \leqq 3500$ K and $1.00 \leqq$ C/O $\leqq$ 50 which include ODF opacities of HCN and C_2H_2, as well as the usual diatomic opacities, has been calculated. We refer to these as the EGJN models (Ericksson et al., 1985) and eagerly await their publication.

Finally, we mention a set of 10 carbon-rich models with the parameters 3000/0.0/S except C; C/O = 1.05, in which the fractional abundance of H was incrementally reduced and replaced by He (Johnson et al., 1985). Interestingly, even this drastic change in composition alters the thermal structure (T–τ) only very slightly, whereas the pressure structure (P–τ) is greatly changed (equivalent to a change in gravity). The energy flux curve is also changed in the sense that, as the hydrogen abundance is decreased, the H^- flux peak at 1.67 μm is steadily reduced relative to the blackbody peak near 1.0 μm.

Again, how well do the model makers agree among themselves? A comparison of models for carbon stars is more difficult than for M stars because the extra dimensions of chemical composition mean less overlap between different sets of models. Nevertheless, in Figure 7-9 we compare, for models with the parameters 3000/0.0/solar except C; C/O = 2.00, the thermal structure ($T(\tau)$) and the pressure structure ($T(P)$) of the HJ (Johnson, 1982) models and EGJN models (Gustafsson, private communication, 1984). For models with diatomic molecular opacities only, both the thermal structure and the pressure structure are in good agreement, the differences being attributable to differences in molecular data and computer codes. When polyatomic opacities are included, the thermal structure is almost unchanged,

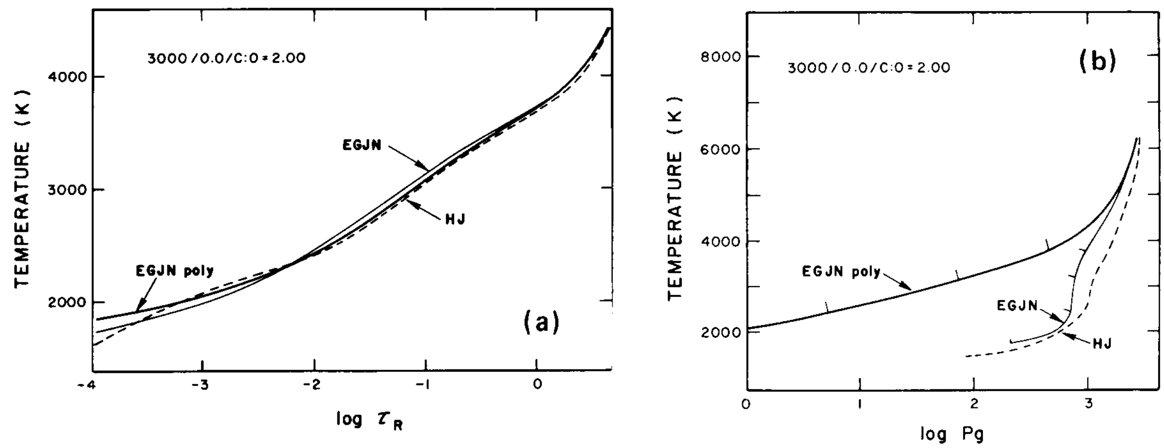


Figure 7-9. Comparison of carbon-rich models computed by the Swedish group (EGJN) and the Indiana group (HJ) and the effect of polyatomic opacities. (a) The T–τ structure of models by the two groups with (EGJN poly) and without (HJ, EGJN) HCN opacity. (b) The T-P structure of models with and without HCN opacity. (Gustafsson, private communiction, 1984; other references are given in the text.)

but the pressure is greatly reduced because of the large increase in opacity.

From this brief overview of models with carbon-rich composition, it should be apparent that the coverage of parameter space is sparce; we are barely beginning. *Before using any model atmosphere* for red-giant stars to calculate quantities, such as profiles of very strong lines, which depend sensitively on the temperatures in the extreme outer layers ($\tau \leq 0.0001$, say), one should *contact the maker of the model* to discuss problems. (The guarantee normally covers only the photosphere!)

SPHERICAL MODELS

While calculations (Auman, 1969; Johnson, 1972; Tsuji, 1976a) confirm the adequacy of PPG for giants ($\log g = 2.0$), it may fail for low-mass supergiants and Miras. Three changes must be made in the usual equations to accommodate sphericity.

1. The equation of radiative transfer must include a curvature term:

$$\mu \frac{\partial I}{\partial r} + \frac{(1-\mu^2)}{r} \frac{\partial I}{\partial \mu} = \epsilon - \chi I ,$$

where $I(r,\mu,\nu)$ is the specific intensity, $\mu = \cos\theta$ as usual, $\epsilon(r,\nu)$ is the volume emissivity, and $\chi(r,\nu)$ is the linear extinction coefficient. Azimuthal symmetry and time-independence are assumed. If $(\partial I/\partial\mu) = 0$, one recovers the usual equation of PPG.

2. The surface gravity is no longer constant, but $g = GM/r^2$, and the variation of g (but not M) with r must be accounted for.

3. A third parameter, in addition to T_{eff} and g, is needed to specify fully the atmosphere. This is often the stellar radius, measured to some reference level, such as $\tau = 1.00$.

The complete equations for spherical geometry and methods for their solution have been available for several years (cf. Mihalas, 1978). Rather different mechanisms produce atmospheric extension in the early-type and the late-type stars. In hot stars, electron scattering produces an outward radiative force which leads to an extended atmosphere. Contrarily, red-giant atmospheres are extended only because of the very low surface gravity (unless, of course, there are outward flows; see Goldberg, this volume).

A commonly used measure of the importance of sphericity is the *extension*, or the relative thickness of the atmosphere compared to the stellar radius. More precisely, the extension is $d = (r(\text{top}) - r(\text{phot}))/r(\text{phot})$, where $r(\text{top})$ is the radius measured to the point at which $\tau = 0.0001$, say, and $r(\text{phot})$ is the stellar radius measured to $\tau = 1.00$. Authors differ slightly in employing a Rosseland optical depth or a monochromatic optical depth and in the exact value of the optical depth (10^{-5} to 10^{-3}) chosen for the top of the atmosphere.

To delineate the regions of the HR diagram in which sphericity might be important, Hundt et al. (1975) and Schmid-Burgk and Scholz (1975) computed a series of spherical models for late-type stars having gray opacities. Of interest here are two principal findings: (1) many red giants and supergiants of moderate mass ($M \approx 1\ M_\odot$) develop extended atmospheres at some epoch during their lives, usually near the peak of their RG luminosity; and (2) temperatures in the outer layers are considerably lower than in a comparable plane-parallel model.

More realistic models were obtained by Watanabe and Kodaira (1978, 1979), who included the line opacities of H_2O, CO, CN, OH — approximated with a mixture of SM, JOA (just overlapping approximation), and VAEBM. In these spherical models, the temperature was lower in the outer layers (by 150 to 200 K in the model with $T_{\text{eff}} = 3200$ K, for example) and higher in the deeper layers than in a corresponding PPG (here called compact) model. The density was also lower in the outer layers than in a corresponding PPG model. The number density of most molecular species, such as CO and CN, followed the overall decrease, but there were certain exceptions, notably those for which the increased efficiency of molecular formation due to the lower temperature overcomes the reduction in the overall density (e.g., H_2O in the model with $T_{\text{eff}} = 3200$ K). For atmospheres with extensions up to $d = 1.5$, these workers found: (1) an increase of atmospheric extent, (2) a reddening of (R–I), and (3) a decrease in the absolute visual magnitude. In addition, stellar diameters measured at wavelengths within those molecular bands formed in extreme outer layers and hence sensitive to atmospheric extension, such as H_2O and TiO, increased as much as a factor of 1.5 compared to the adjacent continuum.

Similar calculations which reinforced these tentative conclusions were carried out by Schmid-Burgk et al. (1981) and Kipper (1982) for models with solar composition. Most of the conclusions were drawn from models computed with continuous opacities and H_2O only, which is probably the dominant absorber in this temperature range. These authors document the great sensitivity of the thermal structure to the treatment of molecular opacities, the incorrect treatment of which can obscure other effects. Figure 7-10 (Schmid-Burgk et al., 1981) illustrates one of the principal effects of sphericity—a lowering of the temperature in the outer layers, where divergence of the flux is important. The temperature drop is greater in the lower gravity A models. (A slight backwarming—too small to be seen here—accompanies the surface cooling.) In the outermost layers, the temperature decrease is 300 to 500 K for a model with extension $d = 0.25$ compared to a compact model. The other principal effect of atmospheric extension—the decrease in density in the outer layers—is illustrated in Figure 7-11 (Schmid-Burgk et al., 1981). The key effect is the drop in pressure (and density as well) from A0 to A2 and B0 to B3; the lower gas density arises from the decreasing gravity, which falls as r^{-2} in these spherical models. Figure 7-11 also shows the radius (r/R) plotted against an optical depth scale ($\tau(1.2\ \mu\text{m})$); the transformation from radius to optical depth is sensitive to the details of the opacities and to the relation of H and H_2, which dominate the

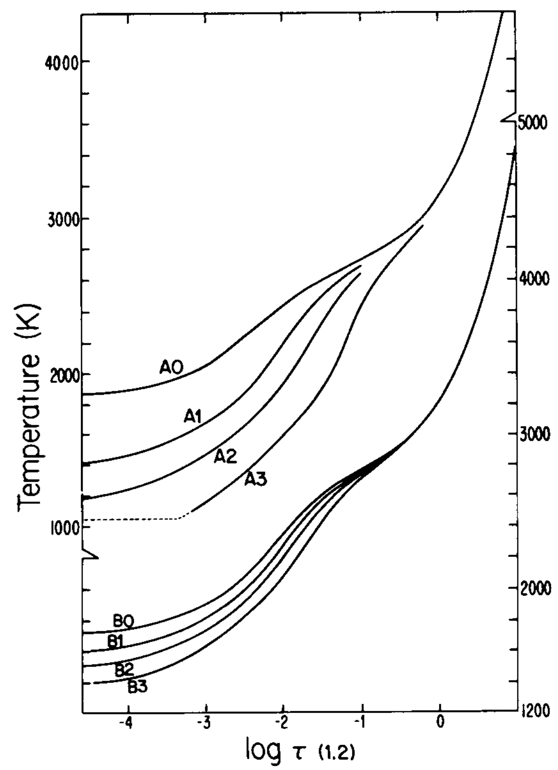


Figure 7-10. The effect of atmospheric extension on the thermal structure of a star. For each series of models, the extension of the atmosphere is increased from A0 (PPG) and B0 (PPG) to A3 (d = 1.39) and B3 (d = 0.20). Parameters of the models are: A: 3000/-1.0; B: 3000/0.0 (from Schmid-Burgk et al., 1981).

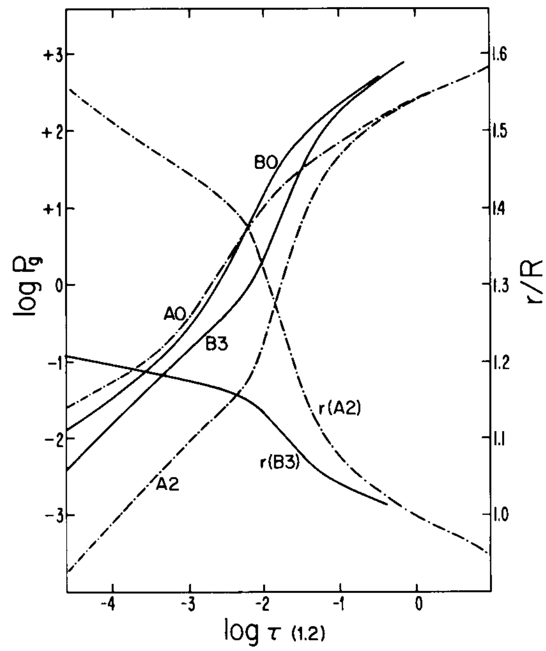


Figure 7-11. Effect of atmospheric extension on the gas pressure for the models of Figure 7-10 (from Schmid-Burgk et al., 1981). The radius (r/R) is also shown for the most extended models.

equation of state at these low temperatures (Schmid-Burgk et al., 1981). As is clear from a consideration of the two figures, the number density or partial pressure of a particular species depends rather critically on the details of the temperature/density relation in the outer atmosphere, which is not well known at this stage of research. Schmid-Burgk et al. also estimate the maximum tolerable value of the extension to be $d = 0.05$ or perhaps as large as $d = 0.10$ in certain cases. The values are exceeded, *at some epoch, in almost all M giants* with luminosities $\geqq 10^3$ solar luminosities and masses near 1 $M_\odot$ (see, for example, Scalo et al. (1978) and Scalo and Miller (1979) for an estimate of these masses).

Pursuing further the consequences of variations in the chemical composition for M supergiants, still within the framework of SM molecular opacities, Wehrse (1981) varied separately the abundances of He, the CNO group, and the metals. Lowering the metal abundance increases atmospheric extension and surface cooling, whereas introducing a hydrogen deficiency reduces sphericity effects. However, because of its interplay of various factors, separating sphericity from compositional effects appears to be extremely difficult.

To find an observational third parameter, Scholz and Wehrse (1982) computed very cool (T_{eff} = 2750, 3000 K) models based on SM opacities of several molecules in addition to H_2O and then calculated band strengths from these models. They find that *for their models*: (1) the 1.04-4.00 and the 2.10-4.00 μm colors can be calibrated to yield effective temperatures; (2) certain metal/ion atomic lines can yield the surface gravity; and (3) TiO lines in the red are especially sensitive to atmospheric extension.

Again utilizing SM opacities, Scholz and Tsuji (1984) study carbon-rich spherical atmosphere. Because of the greater temperature sensitivity of the carbon-rich opacities, these atmospheres are *less extended* than oxygen-rich atmospheres. For the same reason, gas pressure and density are higher in C-rich spherical atmospheres than comparable PPG (compact) atmospheres, but oxygen-rich stars behave *oppositely*.

Spherical models will clearly come to play a major role in the study of red-giant stars—especially for supergiants and Miras—as soon as realistic opacities can be used.

CONVECTION AND INHOMOGENEITIES

Despite much research on the subject, convection is one of the more uncertain aspects of atmospheric modeling. The problem is simply that we do not understand the phenomenon; not only is convection an extremely complex hydrodynamic phenomenon, but we are without the benefit of sure terrestrial or laboratory analogs.

Envelopes of red-giant stars must be the scene of deep strong convection due to the opacity associated with hydrogen and helium ionization/recombination zones. Somewhere in the deep photosphere—near the depth where $\tau(R) = 10$, but depending on effective temperature, surface gravity, and composition—the envelope changes rapidly from a convective to a radiative regime. Photospheres lie at and immediately above this transition, and it has been shown for both M stars (Auman, 1969) and carbon stars (Johnson, 1972) that convection, if treated in the usual local mixing-length theory without overshoot, is unimportant in the photosphere (see also Gustafsson et al., 1975). Because of the rapid changeover from a convective to a radiative regime, our conclusions regarding convection may be sensitive to the details of the photospheric models, and at present, these are somewhat uncertain. Thus, although convection appears to be unimportant, this is not a robust conclusion, particularly if inhomogeneities are present.

Several authors have found it convenient to distinguish convective theories as being "local" or "nonlocal." The local theories are constructed so that, at a given position in a star, all quantities depend on local values of other variables, while nonlocal theories depend on values of variables at distant points in the atmosphere. By all odds, the most widely used treatment of convection is the local mixing-length theory (Böhm-Vitense, 1958). This theory has the advantage of ease of computation, a free parameter which can, if necessary, be adjusted to match observations, and comparability with past research. In the mixing-length theory (cf. Mihalas, 1978), the convective flux is

$$F_c = \varrho C_P \, v \, T (\nabla - \nabla_E) \, \ell / H \,, \tag{7-5}$$

where v is the average velocity of a bubble of gas, $\nabla = d \ln T / d \ln P$ in the ambient atmosphere, ∇_E refers to the value of ∇ in the convective element, ℓ is the mixing length (the average distance traveled by a bubble before it loses all its energy), and H is the pressure scale height. The average velocity v is found by setting the kinetic energy of the bubble to one-half the work done by the buoyant force. The other half goes to overcome friction by other bubbles. The gradients ∇ and ∇_E are calculated from considerations of the energy loss by the bubble during its lifetime and depend on assumptions regarding optical thickness of the bubble and its rate of radiation. To the present time, all significant sets of atmospheres for red-giant stars have been based on radiative equilibrium, or if convection has been included, it has been handled by the local mixing-length theory. Improvements made in or suggested for the standard mixing-length theory include taking account of the optical depth of the convective bubble (Henyey et al., 1965) and averaging the opacity across horizontal layers, rather than averaging the temperature and then calculating the opacity (Deupree, 1979).

The question of the mixing length, or the ratio of mixing length to pressure scale height (ℓ/H), has provoked considerable study. Most often one simply sets $\ell/H = 1.0$ or 1.5. For cool giant stars, surface convection is of such minor importance that ℓ/H apparently cannot be fixed from observations. (Indeed, it is not even certain whether convection can be treated by a local theory.) It is possible that the shape of the flux peak at 1.67 μm (Bell et al., 1976b) or the profile of a strong LTE line such as Ca I 6572 or CO (Carbon et al., 1976) might be a valuable diagnostic feature for temperature structure and hence convection. In a different approach toward determining the scale length of convection, Böhm-Vitense and Nelson (1976) note the near coincidence in the HR diagram of: (1) the boundary line separating convective and nonconvective stars, and (2) the red edge of the Cepheid instability strip and use this fact to fix the mixing length at H, the pressure scale height. A later study (Böhm-Vitense and Dettman, 1980) demonstrates that chromospheric emission is observed by IUE only for giants cooler than F2, and by implication, this denotes the onset of convection. A comparison of IUE fluxes with model predictions (Böhm-Vitense, 1982) appears to confirm this conclusion.

Based on scaling the quantity ($\nabla - \nabla_{ad}$), which is related to the depth of the convective zone, Schwarzschild (1975) speculated that enormous granules ($\sim 10^8$ km) might exist on Betelgeuse. Although Böhm-Vitense and Nelson (1976) found no evidence for such giant eddies, such features may not be entirely theoretical. High-resolution spectrophotometry of the Ca II K line in α Boo (Chiu et al., 1977) appears to require two states of excitation of the chromosphere. The normal state gives a mass loss $dM/dt < 10^{-9} M_\odot$/yr, whereas the excited states gives $dM/dt \sim 8 \times 10^{-9} M_\odot$/yr. The authors wonder whether the excited state arises from the appearance at the surface of a giant convective element. The argument for a two-component atmosphere is strengthened by a study of CO, which shows that no single-component atmosphere can match observations of both the wings of the Ca II infrared triplet (Ayres and Linsky, 1975) and the CO lines (Heasley et al., 1978). Of greater relevance here is Betelgeuse, whose Ca II and Mg II lines appear to require an inhomogeneous atmosphere (Basri et al., 1981). Finally, observations of a regular variation in the polarization of this object (Hayes, 1984) strongly suggest rotation of a "hot spot" (Schwarz and Clarke, 1984; Clarke and Schwarz, 1984)—perhaps a large convective cell or a magnetic complex.

Nonlocal theories of convection have not yet been widely adopted for cool star atmospheres. It may be, however, that the application of the most successful of these theories will constitute a major field of progress during the coming decade. A pioneering, nonlocal theory of convection (Spiegel, 1963) made an analogy with the equation of radiative transfer, the "source function" being chosen so that, in a uniform medium, this "generalized mixing-length" theory gave the same flux as the standard theory. In a "diffusion approximation", this theory was later applied, with gray opacities, to the Sun (Travis and Matsushima, 1973a) and to G and K dwarf stars (Travis and Matsushima, 1973b). For his study of yellow giants, Parsons (1969) introduced a nonlocal element into the standard mixing-length theory by averaging the convective velocity over the entire trail of the convective elements. Convective thermal plumes in the terrestrial atmosphere provided an analogy for a totally different nonlocal theory (Ulrich, 1970a), which was then applied to the Sun, where significant overshooting into optically thin regions was found (Ulrich, 1970b). A critical review of these theories and numerical comparison of their predictions for the Sun was made by Nordlund (1974). With a correction in the Ulrich theory, its similarity to the schemes of Parsons was demonstrated. The low value of ℓ/H required by the Travis-Matsushima formulation to match solar limb darkening was explained as a result of excessive overshoot. By a proper choice of such free parameters as the mixing length, the methods of Parsons, (modified) Ulrich, or even the simple mixing-length theory can be made to yield similar results and may confidently be used to calculate models of the optically thin photospheres of main-sequence stars and giants down to K0. In deeper layers, in which convection is more important, the theories offer qualitatively correct but parameterized descriptions of convection.

Nonlocal theories, particularly those of Spiegel (1963) and Ulrich (1970a, 1970c), were compared by Ulrich (1976), who showed that these differ principally in the way in which the averaging is done. The length over which averages are taken—the diffusion length—thus becomes a significant parameter. The choice of a particular kernal function for the integration leads to the more tractable differential equation found by Travis and Matsushima (1973a, 1973b).

Another approach is the anelastic convective theory (Latour et al., 1976), which borrows from meteorology the idea of greatly restricting the allowed acoustic modes. Horizontal modes are also simplified to permit a more detailed examination of the large-scale vertical structure. In subsequent papers, the anelastic convective theory has been applied to convection in an A star, for which the convective zones are well separated and convection is relatively modest (Toomre et al., 1976; Latour et al., 1981).

All of the foregoing theories, whether local or nonlocal, are based on a "one-stream" model of convection in which convective elements (bubbles or plumes) move relative to a static ambient atmosphere. A more powerful, but much more complex, scheme would be a "two-stream" model. A number of such semi-empirical models have been constructed to interpret velocity and brightness inhomogeneities in the solar photosphere. A more or less deductive two-stream model of convection was derived by Ulrich (1970a, 1970c), and a fully deductive model was derived by Nordlund (1976). This latter method has been applied to convection in the Sun and cooler stars with excellent results (Nordlund, 1980, 1982). It is possible to simulate realistically photospheric granular convection and to predict reasonable values of line shifts and widths, which in turn give rise to what is called macroturbulence and microturbulence. Overshooting of convective elements into superficial radiative layers is specifically allowed, and the resulting large temperature differences (~ 10 to 10^3 K) between the two streams in visible layers of the photosphere constitute one of the most dramatic differences between these results and those of the local mixing length theory. At large depths ($\tau > 50$), the predictions of the two-component theory and those of the local mixing-length theory are quite similar. The interplay of various terms in the energy balance between the hot (ascending) components and the cool (descending) components at various heights is complex and instructive. In the outermost layers, where the radiation begins to escape directly to the surface, these temperature fluctuations are due principally to radiative terms—radiative heating from below, radiative exchange between the two components, and radiative losses to the surface—which implies that the temperature difference in the optically thin layers depends on the horizontal scale of the convecting elements and emphasizes the crucial role of that quantity (Schwarzschild, 1975, Böhm-Vitense and Nelson, 1976). Clearly, there is a hint of far-reaching potential in this two-stream theory, and we eagerly look forward to future advances (Nordlund, 1982, 1985) and applications to red-giant stars.

A fascinating study of the effect of certain improvements in the mixing-length theory and a comparison with other theories has now been made for the Sun (Lester et al., 1982). Two improvements were incorporated into the ATLAS6 treatment of convection: (1) the proper horizontally averaged opacity (Deupree, 1979) was taken, and (2) a depth-variable mixing length was employed. The results are summarized by the authors as follows, in comparing the new calculations with otherwise identical calculations for the Sun without these modifications.

1. Convection (in the improved treatment) transports a smaller fraction of the total flux.

2. The convection zone is narrower, and F_{conv}/F_{tot} is smoother.

3. The temperature is higher in the convective region.

4. The model is brighter in the region 1500 to 2000 Å.

5. Small differences arise in the visual spectrum. These differences result almost entirely from the use of the horizontally averaged opacity. Compared to other theories, there are also differences, and these result from both the horizontally averaged opacity and the variable mixing length (Deupree and Varner, 1980).

6. Compared to nonlocal mixing-length theories, the new calculation has less convection in the upper photosphere, and this leads to these models being hotter below $\tau_R = 0.6$.

7. Compared to Nordlund's (1976) two-stream model, the new results agree well above $\tau = 15$, but are much hotter below that depth.

8. Compared to empirical solar models, there is good agreement above log $\tau = 0.4$, but the new results are too warm below that point.

The Sun is a natural testing ground for new theories, and it may be some time before better theories are applied to the red-giant stars which form the subject of interest here. At present, the entire field of convection is fragmented and in turmoil (as if convecting?), making a coherent review next to impossible (cf. Marcus et al., 1983). Worse yet, for our purposes, application of the available theoretical work is almost exclusively to the Sun, for which high-quality observations of limb darkening, granulation, and line profiles can be obtained and an empirical model is available. Improvements in convective theory for red-giant stars will likely be borrowed from applications in warmer stars. Two comments are relevant here: (1) inhomogeneities are likely to be relatively much more important in red-giant stars than in the warmer more compact stars in which convection is usually studied; and (2) because red giants are variable and thermal models are crude, the power of this combination to constrain or guide the theory of convection is weak.

In summary, we note that convection and inhomogeneities might play important roles in at least the following ways.

1. Because concentrations of certain molecules (H_2O, H_2, HCN, TiO, and CaCl) are extremely temperature-sensitive, their concentrations must increase over cooler regions of the photosphere. One can imagine a feedback mechanism through which increased molecular concentrations further cool the atmosphere (Gustafsson, 1981).

2. The subphotospheric convective zone presumably supplies the mechanical energy to produce a chromospheric temperature rise and perhaps promote mass loss (cf. Schmitz and Ulmschneider, 1981; and de la Reza and Goldberg, both this volume).

3. Grain formation might occur, or occur more readily, in or over cooler photospheric regions (Lucy, 1976; Deguchi, 1980; Draine, 1981, 1984; Woodrow and Auman, 1982; see also Lefèvre, this volume).

4. Microvelocity fields deduced from high-resolution spectra—or their interpretation—would be affected (Dravins et al., 1981; Nordlund, 1982).

5. Departures from LTE might be increased, perhaps dramatically, in an inhomogeneous atmosphere.

6. The conversion of theoretical quantities (such as T_{eff}) to observed quantities (such as colors) might be affected, as has already been noticed for F stars (Nelson, 1980).

INTERLUDE

How well do the predictions of the theoretical models of the previous section compare with the observations of real stars? A moment's reflection suffices to remind us, of course, that these thermal models will yield only a background continuous spectra and symmetric absorption lines, and *there is no possibility that thermal models can reproduce the dazzling array of observations presently available* (as described, for example, by the Quercis in Chapters 1 and 2). Much more work is necessary before we can model, in a self-consistent fashion, such nonstatic and nonthermal phenomena as turbulence, departures from LTE, pulsation, and mass loss (see Figure 7-1). Progress toward these goals is reported in later chapters. It is only fair, however, to give the thermal models a chance to demonstrate whether they can explain even what they were *meant* to explain. In addition, we seek spectral features which are particularly sensitive to a single property of the atmosphere and may therefore help to diagnose the physical and chemical properties of the star. At the outset, we set forth our most difficult problem: how does one test the goodness of a comparison between theory and observation? As will become obvious from several examples, we will stumble over this problem again and again.

SPECTRAL DISTRIBUTION OF ENERGY FLUXES (Photometry, Spectrophotometry, and Colors)

Oxygen-Rich Stars (M and S Stars)

Although the models of Gustafsson et al. (1975) and Bell et al. (1976a) are intended for G and K giant stars, we mention them here because of their obvious implications for cooler stars and the paucity of such comparisons for M, S, and C stars. The excellent overall match of observed spectrophotometry and narrowband photometry with predicted fluxes from these models confirms that the line-blanketing and thermal structure of present models of G–K giants is essentially correct (Gustafsson and Bell, 1979). This impressive work has reached a new level of sophistication for stars of intermediate temperature. Unfortunately, the red-giant stars of interest in this volume constitute a rather different class of stars.

A persistent flux excess shortward of 4000 Å was carefully scrutinized by Gustafsson et al. (1975), who conclude that, although massive chromsopheres, temperature inhomogeneities, and departures from LTE cannot be ruled out as sources for the discrepancy, the most likely interpretation is neglected ultraviolet opacities—from either metal lines, molecular lines, or some continuous absorber. A violet discrepancy is not unique to these models, but has been noted in other models as well. For example, Peytremann (1974) found that his models produced too much flux in the violet, a feature he also suggested was due to insufficient lines there. In her models of F dwarf stars, Böhm-Vitense (1982) finds the same circumstance and suggests that it arises from either an unaccounted-for continuous opacity source or a substantially increased convective flux. A similar problem appears in the M giant models of Johnson et al. (1980). (See Steiman-Cameron and Johnson, 1985.) To investigate the possible luminosity dependence of the relative flux peak at 1.67 μm, for which there is some observational evidence, ODF spectra of CN and CO were computed and found to be in fair agreement with the observations (Bell et al., 1976b). These conclusions are only tentative until additional observations at many frequencies, as well as improved theoretical spectra including the contributions of TiO and H_2O, are available. Limb-darkening coefficients for the set of models have also been calculated (Manduca et al., 1977) for the wavelengths of the *UBV* and *uvby* bandpasses.

Based on Gustafsson et al. (1975) and Bell et al. (1976a) atmospheres with $4000 < T_{eff} < 6000$ K, Bell and Gustafsson (1980) calculate the visual surface brightness (VSB) and study theoretically its dependence on color, gravity, and metallicity. The total effects of all these factors are *smallest* for the V–R color, and the theoretical relation between VSB and V–R agrees well with the empirical relation from stars with known angular diameters (Barnes and Evans, 1976; Barnes et al., 1978; Eaton and Poe, 1984). This agreement provides additional evidence that the background continuous spectrum, including the effects of line blanketing, is well predicted by these theoretical models for G and K giants.

Spectrophotometry provides a somewhat stiffer test of models than does photometry, and the comparison of predictions from the Gustafsson et al. (1975) and Bell et al. (1976a) models with spectrophotometry of G, K, and M stars (Strecker et al., 1979) is of considerable interest. Water vapor, but not TiO, was included in the synthetic spectrum, although neither was included in the calculations of the models. Figure 7-12 shows the comparison for β And (M0 III) and β Peg (M2 II-III) (Manduca et al., 1981); the agreement over most of the range from 1.2 to 5.5 μm is better than 10 percent. (Fits for warmer models are generally even better.) The largest discrepancy in β And occurs in the first overtone bands of CO at 2.5 μm. While that particular problem could in principle be remedied by altering the CNO abundance, the ratio of the fundamental and the first overtone of CO cannot be so easily corrected. The fit for β Peg is slightly worse, but this comparison should not be pushed too far, since the authors suspect that the star may be cooler than any of their models. The authors also note that effective temperatures deduced from model fitting are in satisfactory agreement with other temperature estimates, and angular diameters inferred from fitting at 3.5 μm are in good agreement with measured values of those deduced from the VSB relation and V–R colors.

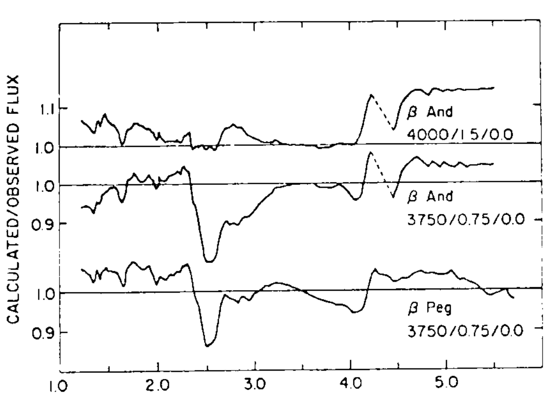


Figure 7-12. Comparison of observed spectrophotometry of early M giant stars β And and β Peg (Strecker et al., 1979) and predictions from the Gustafsson et al. (1975) and Bell et al. (1976a) models identified in the figure (from Manduca et al., 1981).

Comparison of computed fluxes from a grid of SM models (Johnson, 1974) with infrared spectrophotometry of 24 cool giant stars (K0 through M9) taken from above the Earth's atmosphere (Scargle and Strecker, 1979) furnishes some food for thought. As a matter of fact, the predicted fluxes often agree rather well with the observations. Yet some of this agreement must surely reflect the insensitivity of the fluxes to the precise structure of the models because SM models have outer photospheres that are generally too warm. More surprisingly, many of these "models" were interpolated or extrapolated where models did not exist in the original grid. Furthermore, there is a strange clustering of stars with $T_{eff} = 3500$ K, which appears to be an artifact of the available set of models, and the temperatures from several of the models are lower than those found for some of the same stars by other methods (Gustafsson, 1981). All of this is a bit disquieting because it forces us to admit that, unless the effective temperature is constrained rather carefully from other evidence, ***observations, especially for variable stars, can be matched reasonably well by crude models. Better observations***, especially calibrated spectrophotometry of both the visual and infrared regions taken at the same time, *are necessary to test even thermal models.*

Tsuji (1976b) found good agreement between infrared fluxes from his models of M supergiants and accumulated photometry and spectrophotometry of Betelgeuse. The fit was improved if the composition was taken to be C-poor and N-rich, rather than solar. (That the atmosphere of Betelgeuse is actually C-poor and N-rich has recently been determined through a careful study using spectral synthesis (Lambert et al., 1984).) A relatively high microturbulent velocity of 6 km/s was used; this was not taken as evidence for supersonic turbulence, but was attributed to deficiencies in the VAEBM method for molecular line opacities. The best fit to the infrared fluxes (1 to 5 μm) gave an effective temperature of 3900 ± 150 K for Betelgeuse (M2 Iab). The implied angular diameter of 0.40 milliarc-sec is, however, lower than most other determinations. No resolution of this discrepancy was offered, but Tsuji noted that this higher temperature resolves a problem with the K I line (Goldberg et al., 1975; see also Goldberg, 1984). (Could it be that most earlier observations detected the larger radii in TiO bands?) Subsequently, Tsuji (1978b) demonstrated that scattered light from an optically thin dust shell had negligible effect on the *colors*, but it might appreciably increase the measured *angular diameter*, which might resolve the Betelgeuse discrepancy. The strength of the TiO bands (Collins, 1974) implied by the variation of angular diameter of Betelgeuse across them appears to require a lower temperature, however, than 3900 K (Balega et al., 1982). The curious behavior of the angular diameter of Betelgeuse is quite complex and is not yet understood (Goldberg, this volume).

In a pioneering extension of model atmospheres to cooler stars, Tsuji (1978a) saw sufficient agreement between fluxes from his models and available photometry and spectrophotometry of K and M giant stars to define a new scale of effective temperature for these objects. A careful scrutiny of these comparisons reveals that, in the infrared, the slopes of the flux curves of all models are quite similiar, and most of the temperature sensitivity comes from wavelengths less than 1 μm. Here, however, none of the predicted fluxes fit well because of distortions by molecular bands. The fitting is therefore fairly subjective. Nevertheless, if R–I colors are used as a baseline, there is reasonable accord between Tsuji's scale of effective temperature and the scale deduced from the angular diameters then available. Finally, Tsuji pointed out that colors and band strengths could well be understood on the basis of this new temperature scale, but no information about variations in abundances among the CNO elements could be inferred.

As additional values of stellar angular diameters became available, especially from observations of lunar occultations, it became possible to define an empirical temperature scale for red giants from K0 to M6 (Ridgway et al., 1980b). Although the scatter among individual stars is rather high and certain large discrepancies exist even for stars for which the formal errors are small, this calibration of effective temperature against spectral class or Wing color temperature (T_c) has become the criterion against which all theoretical results are tested. From time to time, more values of angular diameters and temperatures have been obtained, but no basic alteration of the Ridgway et al. scale has been necessary.

An additional link between models and observations was forged by Tsuji (1981a), who used his atmospheres and the method of infrared photometry (IR) (Blackwell and Shallis, 1977) to define a new independent temperature scale for M giant stars. Basically, the IR method is a clever application of the relation between observed flux (f_λ), flux emitted at the stellar surface (F_λ), and the stellar angular diameter, in which the monochromatic flux from an appropriate *model* is substituted for the emitted *stellar* monochromatic flux (F_λ) and is used with the *observed* monochromatic flux (f_λ) to obtain the angular diameter. With this value of the angular diameter and the *integrated* observed flux (f), the *integrated* emitted stellar flux (F) and hence the effective temperature can be derived. The derived effective temperature depends only rather weakly on the model used. Furthermore, since the ratio of IR flux to total flux varies roughly as T_{eff}^3, the method is quite powerful in determining the temperature even in the presence of imprecision in the models from missing opacities, inexact opacity treatment, or imperfect match in chemical composition. The necessary equations have been combined into an especially convenient form (Blackwell et al., 1980). From photometric measurements on 62 giant stars from spectral class K1 through M6, Tsuji (1981a) used L-band photometry for the IR fluxes to deduce effective temperatures. The resultant temperature scale agrees quite satisfactorily with that deduced from angular diameters (Ridgway et al., 1980b). A minor difference is that Tsuji found evidence for a plateau at T_{eff} = 3800 K for stars of spectral types M0 through M2 and speculated that, in this spectral range, luminosity might affect the spectral classification in such a way that stars of identical temperature but differing luminosities were assigned different spectral classes. (The reader may detect a minor inconsistency here, because the values of T_{eff} calculated from the models do not show a gravity dependence.) Although a slowing of the steady decrease of T_{eff} with spectral type cannot presently be excluded by angular diameter data, Keenan (1982) has demonstrated that there is a steady progression of color and spectra of Fe, Cr, and TiO lines through these spectral classes.

Another crucial issue, noted but not resolved by Tsuji (1981c), arises from the masses of the M giants inferred from various luminosity measurements and his effective temperatures. In the resultant HR diagram (Figure 12, Tsuji, 1981c) the K–M giants scatter along the theoretical evolutionary track for a star of 3 solar masses, yet present knowledge of the luminosity and initial mass function appears to establish that most of these stars must have masses near 1 solar mass (Scalo et al., 1978; Scalo and Miller, 1979).

Based on the Johnson et al. (1980) models of K and M giants described earlier, Piccirillo et al. (1981) calculated Wing color temperatures and compared the relation between T_c and T_{eff} with that obtained from previous work. The original calibration of T_c against T_{eff} is semiempirical (Ridgway et al., 1980b). As is clear from Figure 7-13, in their respective ranges of overlap, the models of Gustafsson et al. (1975) and Bell et al. (1976a), of Tsuji (1978a), and of the Indiana group (Johnson et al., 1980) agree well with the semiempirical relation. *This agreement of the three sets of theoretical models with the semiempirical relation constitutes the most convincing evidence that theoretical models of early M giants are basically correct.* Piccirillo et al. (1981) further demonstrate that the difference between T_c and T_{eff} is due to a multitude of weak TiO lines, unseen even at fairly high spectral resolution, which depress the stellar flux by significant amounts in the region of the short-wavelength continuum filter at 7540 Å (Wing, 1981).

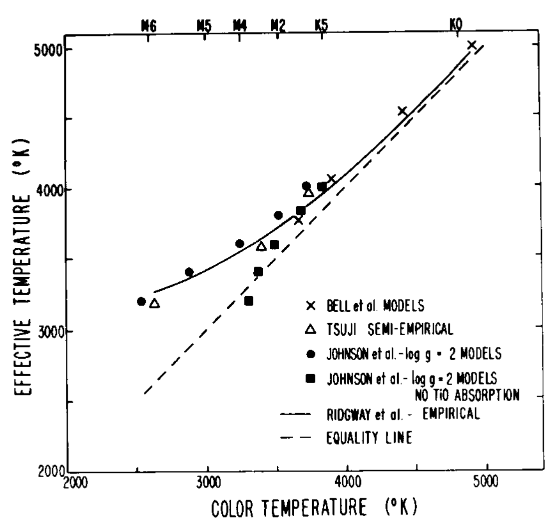


Figure 7-13. The relation between Wing color temperature and effective temperature for K and M giants. Solid line is the semiempirical relation (Ridgway et al., 1980b); symbols represent results obtained from model calculations by three different groups (Gustafsson et al., 1975, and Bell et al., 1976a; Tsuji, 1978a; Johnson et al., 1980). Note that the points calculated without TiO absorption appear to follow the equality line. References are given in the text (from Piccirillo et al., 1981).

Results from the first comprehensive comparison of colors from models of K–M giant stars and observations are also quite encouraging (Steiman-Cameron and Johnson, 1985). Magnitudes on the VRIJHKL broadband system and on the wing narrowband system have been computed for the models of Johnson et al. (1980) and compared to observations. Figure 7-14 shows a comparison of effective temperature against the TiO band strength index computed from the Johnson et al. models, compared with the relations of Ridgway et al. (1980b) and Tsuji (1981a). We note that the Ridgway et al. curve is *semiempirical* (the best fit by eye through the observed points shown), the Tsuji curve is also *semiempirical* since it uses T_{eff} deduced by the IR method and TiO measured for the corresponding spectral classes, and the Johnson et al. curve is *purely theoretical*. The agreement is quite satisfactory, certainly within the large observational scatter, down to a temperature of 3500 K, below which the theoretical TiO band strengths are too weak. Steiman-Cameron and Johnson (1985) have considered several possible effects to explain the discrepancy at lower temperatures, including dissociation of TiO by chromospheric heating, increasing effects of sphericity as the luminosity increases and the surface gravity decreases toward cooler stars, changed thermal structure due to possible carbon enhancement in the cooler stars, and a variation of radius with wavelength. (A chromospheric effect apparently exists (Steiman-Cameron et al., 1985).) In any case, the excellent accord for early M stars is reassuring.

Carbon Stars and S Stars

What of the fascinating peculiar red-giant stars—types N, S, and R? Most model calculations are so recent that time has not allowed careful comparison with observations, and such comparisons remain a pressing need. The assurance we could give earlier that predictions from classic thermal models for early M giants were in reasonable accord with observations of the continuous flux, and that reasonable values of angular diameters and temperatures could be deduced on the basis of model fitting, cannot be repeated for the cooler stars.

The French group (Querci and Querci, 1976) compare two of their triple-alpha-enriched carbon-star models with the observed broadband colors of 19 Psc and UU Aur. The fit for a model with T_{eff} = 3600 K to UU Aur is fair, and the theoretical predictions fit better than the blackbody curve, especially in the 0.8- to 1.6-μm region. The flux curve for 3000 K matches almost as well, but is disfavored by Querci and Querci because of a poor fit to the observed spectrum. All models predict excess fluxes in the blue part of the spectrum. Altogether, the comparison leaves much to be desired because the fit is only fair, and the temperature deduced from the best fit—T_{eff} = 3600 K for UU Aur for the triple-alpha composition—appears to be considerably too high (Ridgway et al., 1980a; Tsuji, 1981b). A lower temperature is obtained for a model based on CNO processing, but synthetic spectra are then in conflict with observations. One is hard pressed, as were Querci and Querci (1976), to select from a large number of choices the specific causes for the relatively poor fits—possibly including at least incorrect chemical composition, neglected opacities, and neglected convection.

In a bold extension of his earlier work on M giants, Tsuji (1981b) used the available models of carbon stars (Querci et al., 1974; Querci and Querci, 1975a) and the L-band observations of Noguchi et al. (1977) to deduce a temperature scale for N-type carbon stars by the IR photometry method described earlier. In this case, it was necessary to correct theoretically the L-band observations for molecular absorption due to HCN and CO. The resulting temperatures are in satisfactory agreement with the six obtained by lunar occultation angular diameters (Ridgway et al., 1980a). Tsuji (1981c) further demonstrates that several broadband colors of carbon stars can be understood on the basis of this effective temperature scale.

Recently, NASA/Ames Research Center spectrophotometry of 10 non-Mira S stars and

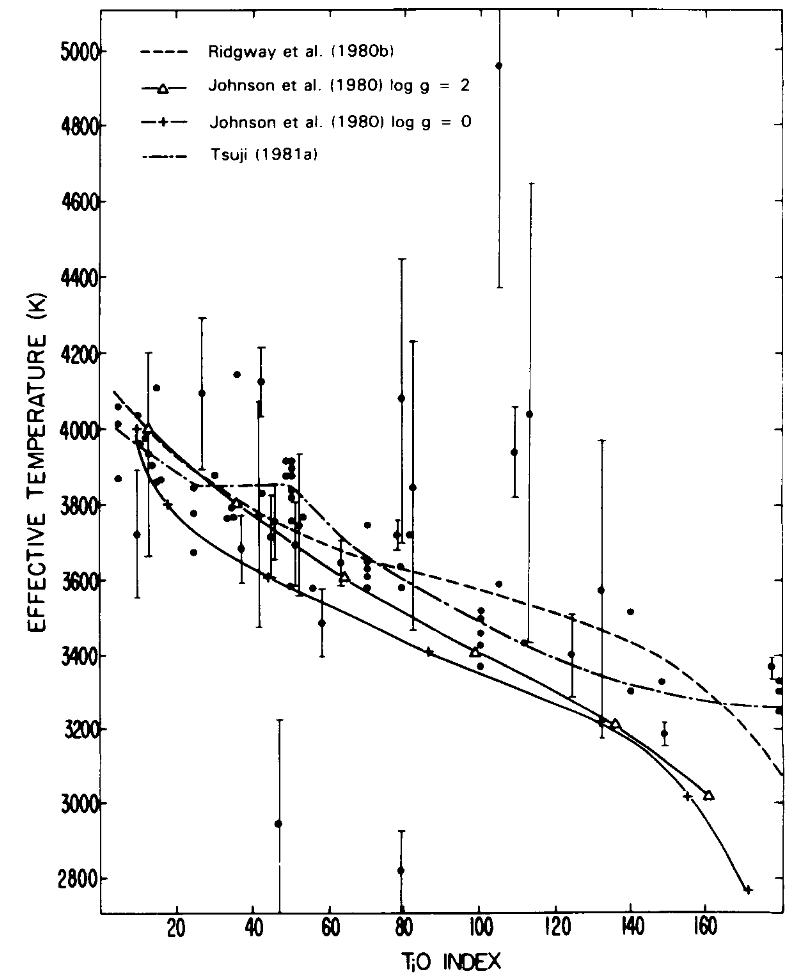


Figure 7-14. The relation between effective temperature and spectral type (TiO band strengths) for K and M giant stars. Shown are the observations, the semiempirical curve which best fits the observations (Ridgway et al., 1980b), the semiempirical relation from IR photometry (Tsuji, 1981a), and the theoretical fit by the models of Johnson et al. (1980). (From Steiman-Cameron and Johnson, 1985.)

fluxes from theoretical models (Johnson, 1982) have been compared in an attempt to calibrate a temperature scale for S stars (Augason et al., 1985). Effective temperatures have been obtained both by model fitting and by the method of IR photometry; results from the two methods are in good agreement. An example of the observed and calculated spectrophotometry for the warm S star, HD 35155, is shown in Figure 7-15. The relative strengths of the fundamental and the first overtone bands of CO do not agree well (a problem we have met before). The keen observer will also note the unusual flat-topped region near the maximum of the H^- peak at 1.67 μm, which differs strikingly in appearance in S and C stars. If the observations were extended to shorter wavelengths, an excess of flux in the visual and ultraviolet spectral regions from the model would also be noticed, and comparisons of the theoretical flux curves to the observations of cooler S stars are not as good as the fit shown. Also, ZrO is not included in these models, and ZrO bands are prominent in the comparison. Although judgment is left to the reader, it is clear that progress is being made in understanding even these complex objects by thermal models.

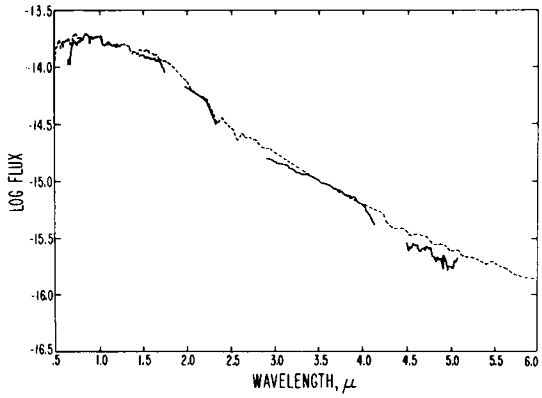


Figure 7-15. Comparison between spectrophotometry of an early S star and predicted flux from an Indiana model (from Augason et al., 1985).

A similar study has been made of the carbon star, Y CVn (C5,5), based on extensive photometry (Goebel et al., 1980). A good fit to the entire spectral range 0.5 to 5.0 μm is obtained for theoretical fluxes from models (Querci et al., 1974; Querci and Querci, 1975a). Not only is the continuous spectral energy distribution well matched, but the molecular band strengths of CN and C_2 are satisfactory. Whether the effective temperature is correct, is not yet clear, however, and one must be cautious of extrapolating general conclusions from an isolated case.

Broadband photometry, which is often the only data available on a given star, carries a minimum amount of information, and the power of observations of one star to constrain the parameters of models is very weak. The power of photometric observations of many stars with differing properties to constrain a series of models is considerably enhanced. Narrowband photometry and spectrophotometry are considerably more powerful tools. Although we await eagerly such complete spectrophotometry for a wide variety of M, S, and C stars, we must be cautious, because unlike warmer stars, essentially all stars of interest here are *variable*. Combining observations taken at random phases—sometimes of necessity—introduces a significant uncertainty into the results.

We conclude that thermal models of the classic variety are able to reproduce the continuous flux curves of giant stars of solar composition in spectral classes M0 through M5. Progress is also apparent for S stars, especially the warmer ones. Testing of C-star models is just beginning. Many further improvements on models are necessary. Continually updated opacities are needed. More molecular lines may be needed for familiar molecules (the huge numbers of weak lines), and opacities from additional molecules should be included. For cool M stars, the lack of an accurate opacity for H_2O is perhaps the single greatest impediment to further progress, but VO is also important. In S stars, ZrO may be important. In cool carbon stars, one must add HCN (Eriksson et al., 1984; Tsuji, 1984; Jørgensen, 1985), C_2H_2, and perhaps C_2H, C_3, and SiC_2. Perhaps we should look at SiO, CS, SiS, FeH, HCl, AlCl, and CaCl—the list could be long.

It is also in these coolest M, S, and N stars (especially the M stars, according to Scholz and Tsuji (1984)) that the effect of sphericity is expected to become important as the gravity decreases with the increase in radius during these late stages of stellar evolution. It is encouraging to note the progress being made by the Heidelberg group and the Japanese workers. Some of our fitting or lack of fitting could be profoundly changed as a result of sphericity. However, a gulf still exists here. The model makers who use plane-parallel models have the extensive opacity data and, understandably, continue their zealous research with these models. Surely there is much room for that. The spherical models have generally been converged with relatively crude opacities. It is time for a merging of forces—of those that have the good opacities with those that have the good spherical codes.

SPECTRAL LINES

There are at least three vital questions, the answers to which will be essential as we consider to what extent line profiles from classic thermal models match observations of real stars. How can one judge quantitatively goodness of fit? Can one isolate the specific reason for the lack of perfect fit? What can one deduce regarding physical conditions in the stellar atmosphere from various lines? The questions are especially pertinent to red-giant stars, in which variability is known and departures from plane-parallel geometry, LTE, and homogeneity are expected. Present progress in answering these questions for K giants can be learned from the detailed review of the conference on Arcturus (Trimble and Bell, 1981). Much less work has been done on the red-giant stars. In addition, spectral lines in supergiants are remarkably broad, and considerable macroturbulence must often be invoked to match observations.

An example of attempts to deduce $^{12}C/^{13}C$ ratios in carbon stars will illustrate the problem in answering definitively the foregoing questions. Values obtained for the well-observed N-type star, V460 Cyg, from excellent high-resolution spectra by several methods make an engrossing story: (1) 100, from a pseudo-curve-of-growth and the isointensity method (Fujita and Tsuji, 1977); (2) 9, from the Minnaert semi-empirical formula (Climenhaga et al., 1977); (3) 32, from a curve of growth (Dominy et al., 1978); (4) 24, from spectral synthesis and a model atmosphere (Johnson et al., 1982); (5) 22–29, from a careful reapplication of the Minnaert formula (Little-Marenin and Little, 1984). The precise values are of less importance in this discussion than the fact that crude methods and atmospheres gave results as good (or bad) as those for better atmospheres. We are evidently in the awkward position of admitting that crude model atmospheres can sometimes both produce good fits to fine observations and permit the inference of useful quantities about the star. Apparently, fitting observed spectra is a necessary, but not sufficient, condition for accepting a thermal atmosphere! Clearly one needs tougher tests for atmospheres: broader spectral intervals, lines from several atoms, ions, and molecules and lines spanning a larger range in region of formation (hence temperature). Otherwise, in the absence of fairly strong constraints, goodness of fit to average lines is a very weak criterion for the identity of the model with a real star.

A useful illustration of the problems of deducing stellar parameters from a comparison of observed and synthetic spectra is provided by the pioneering attempt to determine H, C, N, and O ratios in UU Aur and Y CVn (Querci and Querci, 1976). Such comparisons, the best of which is illustrated for UU Aur in Figure 7-16, provide some reassurance that the models generally have a correct thermal structure, but uncertainty in other parameters do not allow a more detailed test. For example, the models shown give $T_{eff} \approx 3600$ K for UU Aur, yet this is much higher than temperatures inferred for similar N-type stars (Ridgway et al., 1980a; Tsuji, 1981b). Compositions enriched in C (called PL in Figure 7-16) fit generally better than those with CNO-cycle compositions (called DE), but line and band f-values are not sufficiently well known to extract more precise information concerning H, C, N, and O abundances. Although the influence of gravity on line profiles is quite small, it increases slightly toward the lower temperatures. Its influence varies somewhat from band to band; for example, CO and CN behave oppositely, and each behaves differently for PL and DE compositions. Because only one value of the turbulence, $\upsilon = 5$ km/s, was used, little information can be deduced from it. Reasonable overall fits to observed spectra of both stars in three spectral regions could be obtained, and the lack of better matches can also be attributed to imprecision of molecular data as to deficiencies in the atmospheric models. We will find other examples of this sort of impasse.

In their comprehensive study, Gustafsson and Bell (1979) compared LTE synthetic spectra from their models for G and K giants to observed spectra for several wavelength bands; the overall fit is fairly impressive.

An attempt at a more objective approach to the comparison of synthetic and observed spectra was applied to five carbon stars (Olson and Richer, 1979) to infer the values of X(CNO), turbulent velocity, and $^{13}C/^{12}C$ by minimizing the residuals between observed spectra and synthetic spectra based on earlier models (Johnson, 1974). Such differences depend critically on accurate values for wavelengths and other molecular parameters, and perhaps the method was pushed a bit beyond the precision of the data. Yet such methods appear to hold considerable promise for the future as better observational and molecular data become available. Our present interest centers on the goodness of the fit. The authors note that their synthetic spectra fit very well the high dispersion (13 A/mm) observations of TX Psc, Z Psc, X Cnc, UU Aur, and Y CVn in the region 7800 to 8300 A for reasonable values of the parameters. Unfortunately, however, no absolute criterion exists for judging goodness of fit. One of the parameters determined by fitting is the microvelocity. The values derived by Olson and Richer (1979) are all subsonic (Vs = 5 km/s) and significantly lower than values obtained by other workers.

We await the results of a more extensive study of $^{12}C/^{13}C$ in approximately 30 N-type carbon stars from infrared Fourier transform spectrometer spectra and synthetic spectra from unpublished atmospheres (Lambert, 1984; Eriksson et al., 1985; Gustafsson, 1985).

One of the more significant results to come from spectroscopy of peculiar red-giant stars concerns the discovery and strength of the quadrupole lines of H_2 in the infrared (cf. Lambert et al., 1973; Hall and Ridgway, 1977; Goorvitch et al., 1980; Bragg et al., 1982; Jennings et al., 1984). The 1-0 S(1) line of H_2 at 2.1 μm—the line most often studied because of its strength and its location in a clear region of

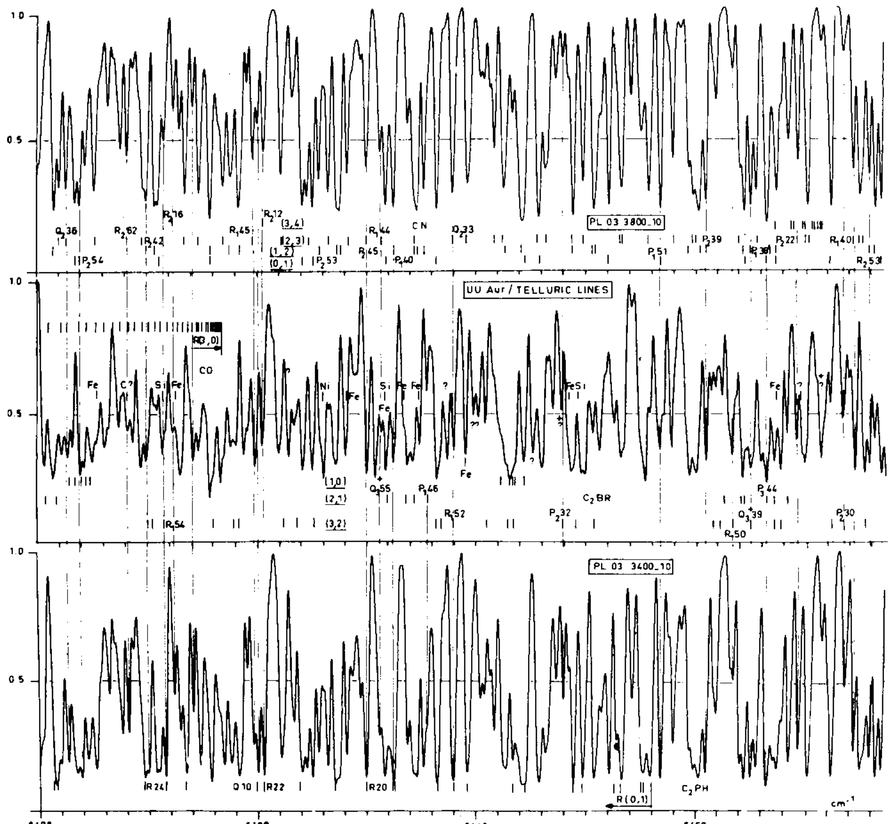


Figure 7-16. Spectra of the carbon star UU Aur and synthetic spectra from theoretical models with compositions representing triple-alpha-enriched material, by the French group (from Querci and Querci, 1976).

the spectrum—has been detected and its strength followed throughout the cycle in the two S Miras, Chi Cyg and R And (Hall and Ridgway, 1977; Hinkle, private communication, 1983). The same line has been detected in the carbon stars, V CrB, U Cyg, RZ Peg, and WZ Cas, but is not present in eight other N-type carbon stars, most of which are irregular or semiregular variables (Johnson et al., 1983). In a parallel investigation, Tsuji (1983) found no trace of this line in any M giant star, even though he studied stars as cool as RZ Ari (M6), WX Vir (M7), and RX Boo (M8). These results are even more unexpected because the line is predicted to be strong by published model atmospheres (Querci et al., 1974; Johnson, 1982; Tsuji, 1978a) in all stars of this temperature range. Several possible causes for the nonappearance of the H_2 line in carbon stars have been suggested (Johnson et al., 1983): (1) the temperature scale for carbon stars is too low by 500 to 800 K; (2) the models are inaccurate—the most important deficiencies being the assumption of a plane-parallel geometry and the neglect of the opacities of polyatomic molecules; (3) H_2 might be dissociated by chromospheric heating; (4) H_2 might be underabundant because of some nonequilibrium process; (5) non-Mira carbon stars might be hydrogen-deficient. Several of these considerations apply to the M giant star as well. At the moment, the source of the discrepancy is unclear. The claim has recently been put forward that the strength of the O–O S(1) line of H_2 is correctly predicted in N-type carbon stars by atmospheres with HCN opacities (Eriksson et al., 1984). The outer atmospheres of cool carbon stars are undoubtedly complex and variable thermodynamic regimes, and we suspect that a full understanding may entail a lot more work.

All of this is not directly relevant for testing thermal atmospheres because only the presence or absence of the line is at stake, and that alone is not a strong criterion for the goodness of the atmosphere. The H_2 quadrupole lines are excellent diagnostics for the temperature and density of the upper photosphere, however, and careful observations of these lines at various phases in several stars are urgently needed.

DIAGNOSTIC FEATURES

Having discussed in some detail the (relatively rare) comparisons of model predictions with observations, we now consider attempts to infer from observations, perhaps with the aid of model atmospheres, specific characteristics of a star. In other words, how can one infer for a particular star the value of a specified property?

An excellent general overview of the strengths and weaknesses of traditional methods of using model atmospheres to infer (or diagnose) values of effective temperatures and surface gravity for stars of all types is that of Pecker (1973). Curves of growth are frequently used, and many good discussions of these are available, including both general references (Aller, 1960; Gray, 1976; Mihalas, 1978) and specific discussions of the use of curves of growth with atmospheres. Required reading for this section is the comprehensive and sobering review of progress in determining effective temperature, surface gravity, microturbulent velocity, chemical composition, and mass for Arcturus (Trimble and Bell, 1981). Uncertain as these quantities are for Arcturus, they are far less certain for red-giant stars, for which observations are much less complete or accurate. Furthermore, red-giant stars are *variable*. Alksne and Ikaunieks (1981) have summarized some of the problems for carbon stars.

Effective Temperature

Effective temperature determinations for red-giant stars will continue to be based, into the foreseeable future, on measured angular diameters and integrated flux curves. Angular diameters are currently best obtained for the red-giant stars from lunar occultation (cf. Ridgway et al., 1980b), and this will continue to be the case for some time. However, interferometers operating in the infrared and later in the visual—perhaps from space—will eventually be developed, leading to a new breakthrough in our knowledge of stellar atmospheres. Although the concept of effective temperature will lose some of its meaning for the coolest and most extended stars if their radii are found to depend on wavelength (Bonneau et al., 1982; Balega et al., 1982), the effective temperature will continue to be used. Because uncertainty in the temperature can mask several other problems, including bad models, its determination is a matter of prime importance. We commend those who have tirelessly pursued this research over the years and encourage them to continue their efforts, improving them when possible by observing at several wavelengths.

The uncertainties of both the *lunar-occultation method* and the reduction of such data have been carefully described (Ridgway et al., 1980a). We reiterate here a warning for the observations of red-giant stars, most of which are *variable.* Broadband colors of the cool carbon star, AQ Sgr, measured as part of a major photometric program (Walker, 1979), for example, are noticably different from remeasurements of the same star at the time of a lunar occultation (Walker et al., 1979). For determining effective temperature, therefore, the photometry for these variable stars should be obtained *at the same time as the angular diameter.*

Effective temperatures have been obtained by the method of *infrared photometry* (Blackwell and Shallis, 1977; Blackwell et al., 1980) for both M giants (Tsuji, 1981a) and cool carbon stars (Tsuji, 1981b). Values for T_{eff} deduced in this manner are in good agreement with those obtained from angular diameters as described earlier. As infrared spectrophotometry becomes available, temperatures can be obtained from an application of the IR method at several wavelengths, thus reducing any possible error. Such a project should be given high priority since it can be done with existing equipment and models.

A paper which calibrates the relation of the *visual surface brightness* (*VSB*) to (V–R) color and claims to extend this relation to stars of all spectral types and luminosity classes (Barnes et al., 1978) has already been noted (see also Eaton and Poe, 1984). Although primarily conceived to yield angular diameters, the method also gives effective temperatures when bolometric corrections are available. The validity of the method for the red-giant stars of this volume has not been securely established because of the paucity of angular diameters and the variability of these stars. The VSB method has been used to determine effective temperatures for R stars (Dominy, 1984) and has been reexamined for N-type carbon stars (Johnson, private communication, 1984).

While *model fitting* is commonly used for deducing effective temperatures of hot stars, its application to red-giant stars has been delayed by the scarcity of both realistic models and infrared spectrophotometry. A single published example employs the N-type carbon star, Y CVn, for which a temperature of T_{eff} = 2600 K was inferred by fitting fluxes from model atmospheres (Querci et al., 1974) to the observed infrared flux curve (Goebel et al., 1980). Effective temperatures for 10 non-Mira S stars—ranging from 2800 to 3600 K—have also been obtained from infrared spectrophotometry by both model fitting and IR photometry (Augason et al., 1985). As additional fluxes from (present and future) theoretical models are calculated and more extensive spectrophotometry becomes available, model fitting should prove to be a powerful tool for deducing effective temperatures.

Color temperatures, such as (R + I) – (J + K), calibrated by comparison with M giants (Mendoza and Johnson, 1965) or by blackbodies (Scalo, 1976; Walker, 1980), have been used for carbon stars, and certainly have their utility today in the absence of more fundamental calibrations. Narrowband photometric colors (cf. Baumert, 1972) may yield more precise temperatures. Additional insights on the relation of colors and effective temperatures for carbon stars is given by Alksne and Ikaumieks (1981), Tsuji (1981c), and Alksne et al. (1983). All of these authors find broadband colors to be generally well correlated with effective temperatures deduced for cool carbon stars. An opposing view, based on studies of carbon stars in our Galaxy and the Magellanic Clouds (Cohen et al., 1981), holds that infrared broadband colors are more indicative of composition than of effective temperature. What is really needed, of course, is a calibration of either broadband or narrowband colors (Ridgway et al., 1980b; Wing, 1981) in terms of effective temperatures for *all types* of red-giant stars, so that these colors may be conveniently used to gage the effective temperature.

A comprehensive scheme for determining the atmospheric parameters necessary for an abundance analysis of G and K giant stars by spectral synthesis or by equivalent widths and a fine analysis, both depending on good model atmospheres, has been described in several papers (Luck, 1979; Luck and Bond, 1980). After a microturbulent velocity has been chosen from a curve of growth, the effective temperature is fixed at the value so that the Fe abundance derived from both Fe I and Fe II lines shows no dependence on the excitation potential of the lower level of the atom. The estimated error in the temperature determined by this means is about ± 0.05 in $\theta = 5040/T$ for early M giants, or about 150 K (Luck and Bond, 1980). No Fe II absorption lines appear in the spectra of M giants, however, and therefore the temperature determinations must be based solely on lines of Fe I. Observational evidence for departures from LTE in the K0 giant star, Pollux, has been presented by Ruland et al. (1980), and the extra precautions against using lines from low-lying levels of Fe I is described by Luck (1982).

A further instructive example is the detailed discussion of the derivation and uncertainty in effective temperature, surface gravity, abundance, and turbulent velocity for giant stars in globular clusters (Pilachowski et al., 1983), some of which are only slightly hotter than the red-giant stars of interest in this volume. The task is, of course, considerably easier in the globular cluster stars, for which a common mass ($0.8\ M_\odot$) can be assumed with less resultant error in surface gravity ($\Delta \log g = 0.25$) than the value deduced from ionization equilibrium ($\Delta \log g = 0.50$). Departures from LTE appear to be the probable cause of the difference in Fe abundance deduced from Fe I and Fe II. If so, the effect is an overionization of Fe I, the direction predicted theoretically (Auman and Woodrow, 1975). Observational evidence for overionization of Ca in M supergiants has been presented by Ramsey (1981).

Effective temperatures for M giants derived from lunar-occultation angular diameters (Ridgway et al., 1980b) or from IR photometry (Tsuji, 1981a) and for M supergiants derived from a variety of sources have been plotted against the unreddened V–K color, and that relation has been used to infer the effective temperature for individual stars for which the (unreddened) V–K colors can be obtained (Luck and Lambert, 1982).

Thermal Structure

Even when the effective temperature of a star is known, the thermal *structure*—$T(\tau)$, $T(P)$, or $T(r)$—is a quantity of additional interest. In the Sun, this relation is easily determined by limb-darkening measurements, but such measurements are essentially impossible in stars, except perhaps in special cases (cf. the discussion of limb darkening for Aldebaran by Ridgway et al., 1982). Possible methods for probing the structure of a stellar atmosphere are few: (1) a flux scan at a fixed wavelength across the solar or stellar disk, (2) a flux scan in wavelength from line center to wing across a strong spectral line, (3) observations at line center of different lines of a multiplet, (4) a flux scan in wavelength from the visible part of the spectrum toward shorter or longer wavelengths, and (5) the variation in angular diameter with wavelength. The fourth method probes increasingly shallower layers in the atmosphere because the opacity increases in both directions due to atomic lines and Rayleigh scattering at shorter wavelengths and increasing free-free absorption at longer wavelengths.

As far as the author is aware, no attempt has yet been made to deduce empirically, using these or any other methods, the thermal structure of even a single red-giant star. All of our knowledge comes from the opposing direction—comparing line profiles predicted from models with those observed—in which the fits can be said to be good (for example, see Querci and Querci, 1975a; Olson and Richer, 1979; Johnson et al., 1982). However, these fits are not good enough, nor is the physics of line formation sufficiently well applied, to be used for diagnostics. We can perhaps say that the models are not *badly* off in thermal structure; as we have seen, however, even crude models of atmospheric structure and line formation yield profiles in good agreement with observations.

Ramsey and Johnson (1975) attempted to find sufficiently rich and strong multiplets to apply method (3) and invert their line-center fluxes to find a run of excitation temperature with depth (Ramsey, 1977). Unfortunately, the presence of both rotation and (large) macroturbulence in real red-giant stars places severe restraints on the utility of this method (Desikachary and Gray, 1978).

A study of the strong infrared lines of the first overtone bands of CO in Arcturus (K2 III) reveals an apparently irreducible discrepancy between models deduced from CO and those deduced from Ca II K, forcing a realization that inhomogeneous atmospheres are upon us (Ayres and Linsky, 1975; Heasley et al., 1978). An investigation of these lines in red-giant stars appears to hold considerable promise for deducing thermal structures.

Of particular interest is the value of the temperature minimum. In the Sun, one sees radiation from the region of the temperature minimum in the ultraviolet (1600 Å) and infrared (130 μm) wavelength regions (cf. Vernazza et al., 1981; Avrett, 1984). We have almost no knowledge of this quantity for any red-giant star except Betelgeuse (T_{min} = 2700 K), and even this value is uncertain because of the failure of any homogeneous chromosphere to match both Ca II and Mg II line profiles (Basri et al., 1981). Although Johnson and O'Brien (1983) speculate that the spectral region near 2850 Å is formed at the temperature minimum in three N-type carbon stars, no confirmation by extensive line identification nor atmospheric modeling has yet been published. *We anxiously await the first identification of the wavelength region corresponding to the temperature minimum in any red-giant star.*

Chemical Composition

Much of the interest in red-giant stars arises from the unusual composition of their surface layers. Changes in the composition during the star's lifetime are vital clues to the processes of nucleosynthesis and mixing which have taken place. These processes have been repeatedly discussed from the point of view of the interior of the star (cf. Iben and Renzini, 1983, 1984). Our interest here is in inferring the abundances from stellar lines. In warmer stars, there is usually no problem in finding sufficient atomic and molecular lines to identify and find the abundance of numerous elements, including even those rare species. In cool stars, the problem is much worse, and one must be careful to select those lines which give a valuable diagnosis of abundance. The lines and methods are carefully described in several papers (Wallerstein, 1973; Lambert, 1985; Gustafsson, 1985) to which readers are referred.

Some of the difficulties involved in abundance analysis of the cool giants are described by Luck and Lambert (1982) in their attempt to extract the Li abundance in 31 M giants and supergiants. Even after careful calculation, including spectral synthesis of TiO lines over a 1000 Å spectral band, the level of the continuum near the Li I line remains the largest uncertainty in their analysis.

The well-known difficulties in diagnosing the chemical composition of carbon stars are dramatically illustrated by V460 Cyg (earlier called DS Peg). Repeated analyses of high-dispersion spectra by empirical pseudocurves of growth (curves of growth using central depths rather than equivalent widths) and the isointensity method led to values of $^{12}C/^{13}C$ = 100 (Fujita and Tsuji, 1977). Parallel analyses with similarly high-resolution spectra using the line-depth method and empirical pseudocurves of growth by another group yielded $^{12}C/^{13}C = 8$ (Climenhaga et al., 1977). Later work aimed at reducing the discrepancy gave values of 32 from CN and CO (Dominy et al., 1978), of 25 from spectral synthesis of CN from the same spectra used by Climenhaga et al. (Johnson et al., 1982), and of 22 to 29 from the same method as Climenhaga et al. but with different spectra (Little-Marenin and Little, 1984). Recent work based on synthetic spectra and new model atmospheres which include polyatomic opacities will provide more reliable values of CNO abundances and isotopic ratios in N-type carbon stars (Gustafsson 1985: private communication).

Surface Gravity

Surface gravity not only is important in its own right but also is useful as a constraint on the mass, especially since stars of all masses tend to mingle in that part of the HR diagram where red giants are found. The emergent flux from a plane-parallel model is very weakly dependent on the gravity, and excellent observations for stars for which other parameters (such as chemical composition and turbulent velocity) were well known would be necessary to give any leverage on the gravity. That the gravity of such a well-observed cool giant star as Arcturus should be sufficiently controversial to justify an international conference to investigate the matter testifies eloquently of our ignorance (cf. Trimble and Bell, 1981). The uncertainty in gravity increases as the temperature decreases.

Gravities cannot be directly measured, but must be inferred from some other quantity, such as the pressure. A common method is to adopt for the gravity that value for which the abundance of a common element, such as Fe or Ti, is the same when two different stages of ionization are employed. Finding sufficient unblended lines from two such stages becomes increasingly difficult in the coolest stars.

An illustration of the methods and problems in deducing the surface gravity comes from slightly hotter stars, in which a comprehensive scheme for deducing the effective temperature, surface gravity, and microturbulent velocity for G and K giant and supergiant stars has been outlined by Luck (1979). All of these parameters must be inferred from Fe lines because only that element is represented by a sufficient number of suitable lines. The inferred abundance of Fe depends naturally on the value of all three of these parameters. One proceeds to choose that value of the microturbulent velocity for which there is no dependence on abundance for moderately strong lines ($W \leqq 200$ mÅ). That effective temperature is then chosen for which there is no dependence of the abundance on the excitation energy of the lower level of the line. That value of gravity is then chosen for which the abundance of Fe as deduced from Fe I and Fe II lines was the same. One then iterates as necessary, but this is not usually necessary. Surface gravities in K giant stars deduced by this scheme have estimated uncertainties of a factor of 2.

Unfortunately, because Fe II lines are weak or absent in the spectra of M giants and supergiants (except for the Fe II emission lines in the ultraviolet, which are probably circumstellar (Boesgaard and Boesgaard, 1976)), even the foregoing scheme is inapplicable, and to the author's knowledge, no application of the method of ionization equilibrium to red-giant stars has been made. In fact, some evidence cautions against it (Ramsey, 1981). The following procedure has therefore been adopted by some workers (Luck and Lambert, 1982; Luck, 1982; and references therein). The luminosity of the star, generally estimated from the Wilson-Bappu effect and a bolometric correction, is combined with the effective temperature and compared with theoretical evolutionary tracks on an HR diagram, from which a mass is inferred. From the mass, effective temperature, and luminosity, the surface gravity is deduced. For M stars, the uncertainty in log g is estimated to be 0.5 corresponding to a factor of 3 in surface gravity or mass.

An estimate of surface gravity can also be obtained from the pressure broadening of the wings of spectral lines—the traditional method in hotter stars. If the abundance is not accurately known (a common problem in red-giant stars), that quantity must be determined from nearby weak lines formed in the same region of the atmosphere as the wings of the strong line to be used for gravity determination (Blackwell and Willis, 1977). The method, equivalent to finding lines on the damping portions of the curve of growth and calculating the damping constant, and hence the gravity from these, has not yet been applied to red-giant stars, but it should be useful if spectral lines meeting the necessary criteria can be found.

A new method, specifically aimed at stars with molecular features, has recently been suggested and applied to Arcturus (Bell et al., 1985). This method attempts to deduce the gravity from the pressure as the latter is manifested through molecular concentrations—in this case, MgH. Again, although this is not a cool star in the sense of the definition of this volume, the method has obvious application.

Analysis of the Ca II K line core and wings holds potential for diagnosing the thermal structure, the turbulent velocity, and perhaps the pressure of cool outer atmospheres, but it has not yet been applied to red-giant stars. The method of formation of the line cores is still a matter of some uncertainty, with several contending theories in the field (cf. Linsky, 1980): (1) the width of the emission core (W_o), measured by Wilson and Bappu (1957) and correlated with the absolute visual magnitude of the star, is formed in the Doppler core of the line and is therefore most sensitive to the turbulent velocities in the atmosphere; (2) the emission core is formed in the damping part of the profile and hence is most dependent on the column mass above the temperature minimum (cf. Ayres, 1979); (3) the line is formed in an isothermal layer (no chromosphere necessary) by the fact of a sharp drop in the source function as it changes to coherent scattering just outside the Doppler core (Basri, 1980); (4) the line is formed by a temperature rise due to an instability caused by infrared CO emission (Ayres, 1981; Kneer, 1983). Clearly the diagnostic value of the line for various parameters depends on the mechanism of line formation, and application must await further elucidation of this mechanism, which may not be the same in every star.

Is this relevant for red-giant stars? Can we expect the same processes to hold here? Several authors have derived empirical relations between the width (FWHM) for the Ca K emission and other characteristics of the star (e.g., one common formulation, based on 55 stars, is $\log W_o = -0.22 \log g + 1.65 \log T_{eff} + 0.10(Fe/H) - 3.69$. Other authors find quite similar expressions (Ayres et al., 1975; Engvold and Rygh, 1978; Cram et al., 1979), including some based on theoretical calculations. Sikorski (1982) has generalized somewhat the relation found by Cram et al., and Glebocki and Stawikowski (1980) have extended it to include more fully the dependence of the line width on the core emission intensity. A similar relation holds for the Mg II h and k lines (Weiler and Oegerle, 1979). Unfortunately, the dependence on surface gravity is very weak and is of little value in stars in which the line itself is weak, as it tends to be in M giants. No application of these possibilities have yet been made in our stars except for Betelgeuse. The lines were measured in a few R stars and were used to find the luminosities (Richer, 1975) and thus confirm that the R stars were giants, whereas the hydrogen-deficient R stars were supergiants. In fact, incredible as it may seem, no measurement, not even detection, of Ca K line cores has yet been made in cool carbon stars.

Turbulence

It is remarkable that, in a conference as recent and comprehensive as the IAU Colloquium 51 on *Stellar Turbulence* (Gray and Linsky, 1980), hardly a word can be found on turbulence in red-giant stars. Understandably, theories must be tested under the simplest circumstances and under conditions closer to those on the Sun, which is the standard astrophysical laboratory, but the paucity indicates our present poor knowledge of these processes in cool giants and supergiants.

Methods for deducing turbulence are the classic ones, in which an extra broadening parameter beyond the thermal broadening is added to the line-absorption coefficient to match the stellar line profiles. These methods are described under surface gravity because the two are inferred from the same data. Among early K stars, the macroturbulent velocities appear to remain near 3 km s^{-1} for stars with a range of M_{bol} from +4 to -1, whereas for brighter stars, there is a sharp increase to about 6 km s^{-1} but with considerable scatter (Smith and Dominy, 1979). Values are in the range 2.5 $\pm$ 0.5 km s^{-1} for K and M giants (Luck and Lambert, 1982) to 4.0 $\pm$ 1.0 km s^{-1} for M supergiants (Luck, 1982). Although values for individual stars, especially some supergiants, are somewhat larger than these mean values, they are generally considerably lower than those given in the tabulation by Glebocki (1973), at least partly due to the lower value of turbulent velocity now accepted in the Sun.

In a study of the lithium abundance in 19 G and K Ib stars, microturbulent velocities of 1.9 to 2.6 km s^{-1} and macroturbulent velocities of 4.0 to 9.0 km s^{-1} were determined (Luck, 1977a, 1977b, 1978).

Very few reliable determinations of turbulent velocities in S or C stars have been carried out, but we note that, in a study of two N-type carbon stars by spectral synthesis and model atmospheres, a microturbulent velocity of 4 to 5 km s^{-1} and a weakly determined macroturbulent velocity of 5 km s^{-1} were required to match the broad lines of the CN 2–0 system in two N-type carbon stars (Johnson et al., 1982).

With values of turbulent velocities are now available for several red-giant stars, these have been derived from a variety of methods and data and are not of the same precision. Turbulent velocities from Miras vary with phase, wavelength, and excitation potential because of their formation in different regions of an extended and mass-losing atmosphere. Many of the velocities in the literature are in fact supersonic; the sound speed in a red giant of T_{eff} = 3000 K is about 5 km/s, and it varies little for the conditions described here. Certainly no claim should be advanced for supersonic turbulence on the basis of any values inferred to the present, because deficiencies of various kinds in models and line formation can manifest themselves as microturbulence. There is also a downward trend of the values as spectra of higher resolution and better models have been employed.

The turbulent pressure enters the equation of hydrostatic equilibrium through its contribution to the total pressure. In a sense, it can be thought of as a levitating force slightly reducing gravity. As far as the author is aware, no attempts have been made to deduce the effects of turbulent pressure in cool giants. Indeed, these depend on the same techniques as those for inferring the gravity and atmospheric extension, and any information on turbulent pressure must await more precise measurement of these other quantitites.

Sphericity

Although sphericity may be important in many of the coolest red-giant stars (especially in supergiants and Miras), insufficient knowledge of the influence of other factors makes it difficult to identify any observational features that are dependent primarily on extension. Lines of TiO in the red have been suggested as especially sensitive to atmospheric extension (Scholz and Wehrse, 1982). This area of research is just beginning.

Chromospheres, Nonradiative Heating, Departures from LTE, and Mass Flow

It was envisioned, when this volume was being planned, that we would now proceed to a discussion of those spectral features inexplicable by thermal models, which, therefore, were diagnostic for the nonthermal and nonstationary processes that form the main thrust of this series of books (cf. Thomas, 1983). As has become painfully clear from at least this chapter, however, *even common observations are still beyond the explicatory powers of present thermal atmospheric models.*

Quasi-thermal models, including chromospheres and departures from LTE, are analyzed in a succeeding chapter by de la Reza. As in K giant stars, the best diagnostic features for stellar chromospheres are emission cores in the Ca II H and K lines, Fe II emission lines in the ultraviolet (Bidelman and Pyper, 1963; Boesgaard and Boesgaard, 1976), emission or exceptionally weak or strong absorption in Hα, and emission in Mg II h and k lines (cf. Linsky, 1980, 1982; Baliunas, 1984; Brown, 1984). Lines from coronal and transition regions are generally not seen in red-giant stars because these stars all lie on the "mass-loss" side of the line which divides stars with coronae from stars with mass loss (Linsky and Haisch, 1979; Ayres et al., 1981). The prime diagnostic for mass loss is the violet-shifted central absorption component in the Mg II h and k lines (Stencel and Mullen, 1980). This latter requires a high-resolution spectrum, and most types of red-giant stars are too faint for IUE.

Spectra of M giant and supergiant stars almost universally show all the foregoing indicators. However, except for the well-studied star Betelgeuse (Goldberg, 1984, and this volume), models for the upper photosphere, the region of the temperature minimum, and the chromosphere are generally yet in the future (de la Reza, this volume).

Much less is known about S stars because they have not been so intensively studied. An early study found variable Mg II h and k emission in the S-Mira, Chi Cyg (Cassetella et al., 1980), and emission in this line (h and k are blended at the low resolution needed to detect the ultraviolet emission in these cool objects) has been reported in several non-Mira S stars but with a wide range in strength (Johnson et al., 1984). No high-resolution ultraviolet spectrum has been published.

Certain N-type carbon stars have been known to show Fe II emission lines in the violet (Bidelman and Pyper, 1963), but no Ca H and K line data are available (Richer, 1975). Ultraviolet spectra taken by IUE show weak Mg II h and k lines in the few cool carbon stars that are bright enough to detect (Johnson and O'Brien, 1983). A single high-resolution spectrum of TX Psc is now available and under study. The emission lines in HD 20234 are also variable (Querci and Querci, 1985).

R stars show Ca II H and K emission cores (Richer, 1975), indicating a chromosphere, but low-resolution IUE spectra are remarkably similar to those of G and K giants. The early R stars (R0–R3) show, on low-resolution IUE spectra, Mg II and Mg I in absorption and no emission features whatever. Middle R stars (R5) show either no Mg II h and k line or faint absorption features. Late R stars (R8) show weak Mg II emission features similar to those of N-type stars (Eaton et al., 1985). Only one high-resolution IUE spectrum has been obtained (the R0 HdC star, HD 182040), and this shows only the faintest hint of any emission at h or k.

Chromospheric modeling for red-giant stars had hardly begun (cf. Avrett and Johnson, 1984) and presents a perfect illustration of a field ripe for research (de la Reza, this volume).

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REFERENCES

Alexander, D.R., and Johnson, H.R. 1972, *Astrophys. J.*, **176**, 629.

Alexander, D.R., Johnson, H.R., and Rypma, R.L. 1983, *Astrophys. J.*, **272**, 773.

Alksne, Z., Alksnis, A., and Dzervitis, U. 1983, in *Properties of the Carbon Stars of the Galaxy* (Riga: Zinatne), p. 162.

Alksne, Z.K., and Ikaunieks, Y.Y., 1981, in *Carbon Stars* (Tucson: Pachart), p. 57.

Aller, L.H. 1960, in *Stellar Atmospheres*, ed. J.L. Greenstein (Chicago: Univ. Chicago Press), p. 156.

Athay, R.G. 1972, *Radiation Transport in Spectral Lines* (Dordrecht: Reidel).

Athay, R.G. 1981, in *The Sun as a Star*, ed. S. Jordan, NASA SP-450, p. 85.

Athay, R.G., and Skumanich, A. 1969, *Astrophys. J.*, **155**, 273.

Augason, G.C., Bregman, J.D., Johnson, H.R., and Witteborn, F.C. 1985, *Astrophys. J.* (submitted).

Auman, J.R. 1969, *Astrophys. J.*, **157**, 799.

Auman, J.R. and Woodrow, J.E.J. 1975, *Astrophys. J.*, **197**, 163.

Avrett, E.H. 1984, in *Chromospheric Diagnostics and Modeling*, ed. B.W. Lites (National Solar Observatory).

Avrett, E.H., and Johnson, H.R. 1984, in *Cool Stars, Stellar Systems and the Sun*, ed. S.L. Baliunas and L. Hartmann (New York: Springer-Verlag), p. 330.

Ayres, T.R. 1979, *Astrophys. J.*, **228**, 509.

Ayres, T.R. 1981, *Astrophys. J.*, **244**, 1064.

Ayres, T.R., Linsky, J.L. 1975, *Astrophys. J.*, **200**, 660.

Ayres, T.R., Linsky, J.L., and Shine, R.A. 1975, *Astrophys. J. (Letters)*, **195**, L121.

Ayres, T.R., Linsky, J.L., Vaiana, G.S., Golub, L., and Rosner, R. 1981, *Astrophys. J.*, **250**, 293.

Balega, Y., Blazit, A., Bonneau, D., Koechlin, L., Foy, R., and Labeyrie, A. 1982, *Astron. Astrophys.*, **115**, 253.

Baliunas, S.L. 1984, in *Future of Ultraviolet Astronomy Based on Six Years of IUE Research*, ed. J.M. Mead, R.D. Chapman, and Y. Kondo, NASA CP-2349.

Barnes, T.G., and Evans, D.S. 1976, *Mon. Not. Roy. Astr. Soc.*, **174**, 489.

Barnes, T.G., Evans, D.S., and Moffett, T.J. 1978, *Mon. Not. Roy. Astr. Soc.*, **183**, 285.

Baschek, B., and Scholz, M. 1982, in *Landolt-Bornstein: Numerical Data and Functional Relationships in Science and Technology*, New Series, Vol. 2, Subvol. b: *Stars and Star Clusters*, ed. K. Schaifers and H.H. Voigt (New York: Springer-Verlag), p. 91.

Basri, G.S. 1980, *Astrophys. J.*, **242**, 1133.

Basri, G.S., Linsky, J.L., and Eriksson, K. 1981, *Astrophys. J.*, **251**, 162.

Baumert, J.H. 1972, Ph.D. Thesis, Ohio State University.

Bell, R.A., Edvardsson, B., and Gustafsson, B. 1985, *Mon. Not. Roy. Astr. Soc.*, **212**, 497.

Bell, R.A., Eriksson, K., Gustafsson, B., and Nordlund, Å. 1976a, *Astron. Astrophys. Supplement*, **23**, 37.

Bell, R.A., and Gustafsson, B. 1980, *Mon. Not. Roy. Astr. Soc.*, **191**, 435.

Bell, R.A., Gustafsson, B., Nordh, H.L., and Olofsson, S.G. 1976b, *Astron. Astrophys.*, **46**, 391.

Bidelman, W.P., and Pyper, D.M. 1963, *Pub. Astron. Soc. Pacific.*, **75**, 389.

Blackwell, D.E., Petford, A.D., and Shallis, M.J. 1980, *Astron. Astrophys.*, **82**, 249.

Blackwell, D.E., and Shallis, M.J. 1977, *Mon. Not. Roy. Astr. Soc.*, **180**, 177.

Blackwell, D.E., and Willis, R.B. 1977, *Mon. Not. Roy. Astr. Soc.*, **180**, 169.

Boesgaard, A.M., and Boesgaard, H. 1976, *Astrophys. J.*, **205**, 448.

Böhm-Vitense, E. 1958, *Zt. f. Ap.*, **46**, 108.

Böhm-Vitense, E. 1982, *Astrophys. J.*, **255**, 191.

Böhm-Vitense, E., and Dettman, T. 1980, *Astrophys. J.*, **236**, 560.

Böhm-Vitense, E., and Nelson, G.D. 1976, *Astrophys. J.*, **210**, 741.

Bonneau, D., Foy, R., Blazit, A., and Labeyrie, A. 1982, *Astron. Astrophys.*, **106**, 235.

Bragg, S.L., Brault, J.W., and Smith, W.H. 1982, *Astrophys. J.*, **263**, 999.

Brown, A. 1984, in *Cool Stars, Stellar Systems and the Sun*, ed. S.L. Baliunas and L. Hartmann (New York: Springer-Verlag), p. 282.

Carbon, D.F. 1973, *Astrophys. J.*, **183**, 903.

Carbon, D.F. 1974, *Astrophys. J.*, **187**, 135.

Carbon, D.F. 1979, *Ann. Rev. Astron. Astrophys.*, **17**, 513.

Carbon, D.F. 1984, in *Methods in Radiative Transfer*, ed. W. Kalkofen (Cambridge: Cambridge Univ. Press), p. 395.

Carbon, D.F., and Gingerich, O. 1969, in *Proc. Third Harvard-Smithsonian Conf. on Stellar Atmospheres, Theory and Observation of Normal Stellar Atmospheres*, ed. O. Gingerich (Cambridge: MIT Press), p. 377.

Carbon, D.F., Milkey, R.W., and Heasley, J.N. 1976, *Astrophys. J.*, **207**, 253.

Cassatella, A., Heck, A., Querci, F., Querci, M., and Strickland, D.J. 1980, in *Proc. Second European IUE Conference*, ESA SP-157, p. 243.

Chandrasekhar, S. 1935, *Mon. Not. Roy. Astr. Soc.*, **96**, 21.

Chiu, H.Y., Adams, P.S., Linsky, J.L., Basri, G.S., Maran, S.P., and Hobbs, R.W. 1977, *Astrophys. J.*, **211**, 453.

Clarke, D., and Schwartz, H.E. 1984, *Astron. Astrophys.*, **132**, 375.

Climenhaga, J.L., Harris, B.L., Holts, J.T., and Smolinski, J. 1977, *Astrophys. J.*, **215**, 836.

Cohen, J.G., Frogel, J.A., Persson, S.E., and Elias, J.H. 1981, *Astrophys J.*, **249**, 481.

Collins, J.G. 1974, Ph.D. Thesis, Indiana University.

Cram, L.E., Krikorian, R., and Jefferies, J.T. 1979, *Astron. Astrophys.*, **71**, 14.

Deguchi, S. 1980, *Astrophys. J.*, **236**, 567.

Desikachary, K., and Gray, D.F. 1978, *Astrophys. J.*, **224**, 1073.

Deupree, R.G. 1979, *Astrophys. J.*, **234**, 228.

Deupree, R.G., and Varner, T.M. 1980, *Astrophys. J.*, **237**, 558.

Dominy, J.F. 1984, *Astrophys. J. Supplement*, **55**, 27.

Dominy, J.F., Hinkle, K.H., Lambert, D.L., Hall, D.N.B., and Ridgway, S.T. 1978, *Astrophys J.*, **223**, 949.

Draine, B.T. 1981, in *Physical Processes in Red Giants*, ed. I. Iben and A. Renzini (Dordrecht: Reidel), p. 317.

Draine, B.T., 1984, in *Proc. Conf. on Mass Loss in Red Giant Stars* (Los Angeles: Univ. of California).

Dravins, D., Lindegren, L., and Nordlund, A. 1981 *Astron. Astrophys.*, **96**, 345.

Dumont, S., and Heidmann, N. 1973, *Astron. Astrophys.*, **27**, 273.

Dumont, S., and Heidmann, N. 1976, *Astron. Astrophys.*, **49**, 271.

Dzervitis, U. 1983, *Investigations of the Sun and Red Stars*, **19**, 65.

Eaton, J.A., Johnson, H.R., O'Brien, G.T., and Baumert, J.H. 1985, *Astrophys. J.*, **290**, 276.

Eaton, J.A., and Poe, C.H. 1984, *Acta Astr.*, **34**, 97.

Engvold, O., and Rygh, B.O. 1978, *Astron. Astrophys.*, **70**, 399.

Eriksson, K., and Gustafsson, B., Jørgensen, U.G., and Nordlund, Å. 1984, *Astron. Astrophys.* **132**, 37.

Eriksson, K., Gustafsson, B., Jørgensen, U.G., and Nordlund, Å. 1985 (in preparation).

Faÿ, T.D., and Johnson, H.R. 1973, *Astrophys. J.*, **181**, 851.

Fujita, Y., and Tsuji, T. 1977, *Pub. Astron. Soc. Japan*, **29**, 711.

Gail, H.-P., and Sedlmayr, E. 1984, *Astron. Astrophys.*, **132**, 163.

Glebocki, R. 1973, *Acta Astr.* **23**, 135.

Glebocki, R., and Stawikowski, A. 1980, *Acta Astr.*, **30**, 285.

Goebel, J.H., Bregman, J.D., Goorvitch, D., Strecker, D.W., Puetler, R.C., Russell, R.W., Soifer, B.T., Willner, S.P., Forrest, W.J., Houck, J.R., and McCarthy, J.F. 1980, *Astrophys. J.*, **235**, 104.

Goldberg, L. 1984, in *Cool Stars, Stellar Systems and the Sun*, ed. S.L. Baliunas and L. Hartmann (New York: Springer-Verlag), p. 333.

Goldberg, L., Ramsey, L., Testerman, L., and Carbon, D. 1975, *Astrophys. J.*, **199**, 427.

Golden, S.A. 1969, *J. Quant. Spect. Rad. Transf.*, **9**, 1067.

Goon, G., and Auman, J.R. 1970, *Astrophys. J.*, **161**, 533.

Goorvitch, D., Goebel, J.H., and Augason, G.C. 1980, *Astrophys. J.*, **240**, 588.

Gray, D.F. 1976, *The Observation and Analysis of Stellar Photospheres* (New York: John Wiley Co.).

Gray, D.F., and Linsky, J.L., ed. 1980, in *Proc. IAU Colloq. 51, Stellar Turbulence* (New York: Springer-Verlag).

Gustafsson, B. 1981, in *Physical Processes in Red Giants*, ed. I. Iben and A. Renzini (Dordrecht: Reidel), p. 25.

Gustafsson, B. 1985, *Ann. Rev. Astron. Astrophys.*, in preparation.

Gustafsson, B., and Bell, R.A. 1979, *Astron. Astrophys.*, **74**, 313.

Gustafsson, B., Bell, R.A., Eriksson, K., and Nordlund, Å. 1975, *Astron. Astrophys.*, **42**, 407.

Gustafsson, B., and Olander, N. 1979, *Phys. Scripta.*, **20**, 570.

Hall, D.N.B., and Ridgway, S.T. 1977, *Les Spectres des Molecules Simples au Laboratoire et en Astrophysique, Mém. Soc. Roy. Sci. Liège* (21e Colloque Internationale d'Astrophysique de Liège), p. 243.

Hayes, D.P. 1984, *Astrophys. J. Supplement*, **55**, 179.

Heasley, J.N., Ridgway, S.T., Carbon, D.F., Milkey, R.W., and Hall, D.N.B. 1978, *Astrophys. J.*, **219**, 970.

Henyey, L., Vardya, M.S., and Bodenheimer, P. 1965, *Astrophys. J.*, **142**, 841.

Hundt, E., Kodaira, K., Schmid-Burgk, J., and Scholz, M. 1975, *Astron. Astrophys.*, **41**, 37.

Iben, I., and Renzini, A. 1983, *Ann. Rev. Astron. Astrophys.*, **21**, 271.

Iben, I., and Renzini, A. 1984, in *Physics Reports*, **105**, 329.

Jefferies, J.T. 1968, *Spectral Line Formation* (Waltham, Massachusetts: Blaisdell).

Jennings, D.E., Bragg, S.L., and Brault, J.W. 1984, *Astrophys. J. (Letters)*, **282**, L85.

Johnson, H.R. 1972, in *Proc. Conf. on Red Giant Stars*, ed. H.R. Johnson, J.P. Mutschlecner, and B.F. Peery, (Bloomington: Indiana Univ.), p. 288.

Johnson, H.R. 1973, *Astrophys. J.*, **180**, 81.

Johnson, H.R. 1974, NCAR TN STR-95 (Boulder: NCAR).

Johnson, H.R. 1982, *Astrophys. J.*, **260**, 254.

Johnson, H.R. 1985, in *Cool Stars with Excesses of Heavy Elements*, ed. M. Jaschek and P.C. Keenan (Dordrecht: Reidel), p. 271.

Johnson, H.R., Ake, T.B., and Eaton, J.A. 1985, in *Cool Stars with Excesses of Heavy Elements*, ed. M. Jaschek and P.C. Keenan (Dordrecht: Reidel), p. 53.

Johnson, H.R., Alexander, D.R., Bower, C.D., Lemke, D.A., Luttermoser, D.G., Petrakis, J.P., Reinhart, M.D., Welch, K.A., and Goebel, J.H. 1985, *Astrophys. J.*, **292**, 228.

Johnson, H.R., Beebe, R.F., and Sneden, C. 1975, *Astrophys. J. Supplement*, **29**, 123.

Johnson, H.R., Bernat, A.P., and Krupp, B.M. 1980, *Astrophys. J. Supplement*, **42**, 501.

Johnson, H.R., Goebel, J.H., Goorvitch, D., and Ridgway, S.T. 1983, *Astrophys. J. (Letters)*, **270**, L63.

Johnson, H.R., and Krupp, B.M. 1976, *Astrophys. J.*, **206**, 201.

Johnson, H.R., Marenin, I.R., and Price, S.D. 1972, *Jour. Quant. Spectr. and Rad. Transf.*, **12**, 189.

Johnson, H.R., and O'Brien, G.T., 1983, *Astrophys. J.*, **265**, 952.

Johnson, H.R., O'Brien, G.T., and Climenhaga, J.L. 1982, *Astrophys. J.*, **254**, 175.

Johnson, H.R., and Sauval, A.J. 1982, *Astron. Astrophys. Supplement*, **49**, 77.

Johnson, H.R., and Yorka, S.B. 1985, *Astrophys. J.* (in press).

Jorgensen, U.G. 1985, in *Cool Stars with Excesses of Heavy Elements*, ed. M. Jaschek and P.C. Keenan (Dordrecht: Reidel), p. 191.

Keenan, P.C. 1982, *Pub. Astron. Soc. Pacific*, **94**, 299.

Kipper, T. 1973, *Pub. Tartu Astronomical Obs.*, **41**, 63.

Kipper, T. 1982, *Model Calculations of Stellar Spectra*, Acad. Sciences, Estonian SSR, p. 3.

Kneer, F. 1983, *Astron. Astrophys.*, **128**, 311.

Krupp, B.M., Collins, J.G., and Johnson, H.R. 1978, *Astrophys. J.*, **219**, 963.

Kurucz, R.L. 1970, *Smithsonian Astrophys. Obs. Special Rpt.*, No. 309.

Kurucz, R.L. 1979, *Astrophys. J. Supplement*, **40**, 1.

Kurucz, R.L., Peytremann, E., and Avrett, E.H. 1974, *Blanketed Model Atmospheres for Early-Type Stars* (Washington: Smithsonian Institution).

Lambert, D.L. 1985, in *Cool Stars with Excesses of Heavy Elements*, ed. M. Jaschek and P.C. Keenan (Dordrecht: Reidel), p. 191.

Lambert, D.L., Brooke, A.L., and Barnes, T.G. 1973, *Astrophys. J.*, **186**, 573.

Lambert, D.L., Brown, J.A., Hinkle, K.H., and Johnson, H.R. 1984, *Astrophys. J.*, **284**, 223.

Latour, J., Spiegel, E.A., Toomre, J., and Zahn, J.-P. 1976, *Astrophys. J.*, **207**, 233.

Latour, J., Toomre, J., and Zahn, J.-P. 1981, *Astrophys. J.*, **248**, 1081.

Lengyel-Frey, 1977, Ph.D. Dissertation, University of Maryland.

Lester, J.B., Lane, M.C., and Kurucz, R.L. 1982, *Astrophys. J.*, **260**, 272.

Linsky, J.L. 1980, *Ann. Rev. Astron. Astrophys.*, **18**, 439.

Linsky, J.L. 1982, in *Advances in Ultraviolet Astronomy: Four Years of IUE Research*, ed. Y. Kondo, J.M. Mead, and R.D. Chapman, NASA CP-2238, p. 17.

Linsky, J.L., and Haisch, B.M. 1979, *Astrophys. J. (Letters)*, **229**, L27.

Little-Marenin, I.R., and Little, S.S. 1984, *Astrophys. J.*, **283**, 188.

Luck, R.E. 1977a, *Astrophys. J.*, **212**, 743.

Luck, R.E. 1977b, *Astrophys. J.*, **218**, 752.

Luck, R.E. 1978, *Astrophys. J.*, **219**, 148.

Luck, R.E. 1979, *Astrophys. J.*, **232**, 797.

Luck, R.E. 1982, *Astrophys. J.*, **263**, 215.

Luck, R.E., and Bond, H.E. 1980, *Astrophys. J.*, **241**, 218.

Luck, R.E., and Lambert, D.L. 1982, *Astrophys. J.*, **256**, 189.

Lucy, L.B. 1976, *Astrophys. J.*, **205**, 482.

Manduca, A., Bell, R.A., and Gustafsson, B. 1977, *Astron. Astrophys.*, **61**, 809.

Manduca, A., Bell, R.A., and Gustafsson, B. 1981, *Astrophys. J.*, **243**, 883.

Marcus, P.S., Press, W.H., and Teukolsky, S.A. 1983, *Astrophys. J.*, **267**, 795.

McCabe, E.M. 1982, *Mon. Not. Roy. Astr. Soc.*, **200**, 71.

McGregor, P. 1980, Ph.D. Thesis, Australian Natl. Univ., Canberra, Chapter 5.

Mendoza, V.E.E., and Johnson, H.L. 1965, *Astrophys. J.*, **141**, 161.

Menietti, J.D., and Fix, J.D. 1978, *Astrophys. J.*, **224**, 961.

Mihalas, D. 1969, *Astrophys. J.*, **157**, 1363.

Mihalas, D. 1978, *Stellar Atmospheres* (2nd edition), (San Francisco: Freeman and Co.).

Mihalas, D., and Athay, R.G. 1973, *Ann. Rev. Astron. Astrophys.*, **11**, 187.

Mihalas, D., Kunasz, P.B., and Hummer, D.G. 1976, *Astrophys. J.*, **203**, 647.

Mihalas, D., and Luebke, W.R. 1971, *Mon. Not. Roy. Astr. Soc.*, **153**, 229.

Mould, J.R. 1975, *Astron. Astrophys.*, **38**, 283.

Münch, G. 1946, *Astrophys. J.*, **104**, 87.

Nelson, G.D. 1980, *Astrophys. J.*, **238**, 659.

Noguchi, K., Maihara, T., Okuda, H., Sato, S., and Mukai, T. 1977, *Pub. Astron. Soc. Japan*, **29**, 511.

Nordlund, Å. 1974, *Astron. Astrophys.*, **32**, 407.

Nordlund, Å. 1976, *Astron. Astrophys.*, **50**, 23.

Nordlund, Å. 1980, in *Proc. IAU Colloq. 51, Stellar Turbulence*, ed. D.F. Gray and J.L. Linsky, (New York: Springer-Verlag).

Nordlund, Å. 1982, *Astron. Astrophys.*, **107**, 1.

Nordlund, Å. 1985, *Astron. Astrophys.*, (in preparation).

Olander, P.P. 1981, *Uppsala Astr. Obs. Rpt. No. 21.*

Olson, B.I., and Richer, H.B. 1979, *Astrophys. J.*, **227**, 534.

Parsons, S.B. 1969, *Astrophys. J. Supplement*, **18**, 127.

Pecker, J.C. 1951, *Ann. d'Astrophysique*, **14**, 152.

Pecker, J.C. 1973, in *Proc. IAU Symp. 54, Problems of Calibration of Absolute Magnitudes and Temperature of Stars*, ed. B. Hauck and B.E. Westerlund (Dordrecht: Reidel), p. 173.

Peytremann, E. 1974, *Astron. Astrophys.*, **33**, 203.

Piccirillo, J. 1980, *Mon. Not. Roy. Astr. Soc.*, **190**, 441.

Piccirillo, J., Bernat, A.P., and Johnson, H.R. 1981, *Astrophys. J.*, **246**, 246.

Pilachowski, C., Sneden, G., and Wallerstein, G. 1983, *Astrophys. J. Supplement*, **52**, 241.

Querci, F., and Querci, M. 1974, *Highlights of Astr.*, **3**, 341.

Querci, F., and Querci, M. 1975a, *Astron. Astrophys.*, **39**, 113.

Querci, F., Querci, M., and Kunde, V.G. 1971, *Astron. Astrophys.*, **15**, 256.

Querci, F., Querci, M., and Tsuji, T. 1972, *Mém. Soc. Roy. des Sci. de Liège, 6 Ser.*, **3**, 179.

Querci, F., Querci, M., and Tsuji, T. 1974, *Astron. Astrophys.*, **31**, 265.

Querci, M., and Querci, F. 1975b, *Astron. Astrophys.*, **42**, 329.

Querci, M., and Querci, F. 1976, *Astron. Astrophys.*, **49**, 443.

Querci, M., and Querci, F. 1985, *Astron. Astrophys.*, **147**, 121.

Ramsey, L.W., and Johnson, H.R. 1975, *Solar Phys.*, **45**, 3.

Ramsey, L.W. 1977, *Astrophys. J.*, **215**, 827.

Ramsey, L.W. 1981, *Astrophys. J.*, **245**, 984.

Richer, H.B. 1975, *Astrophys. J.*, **197**, 611.

Ridgway, S.T., Jacoby, G.H., Joyce, R.R., Siegel, M.J., and Wells, D.C. 1982, *Astron. J.*, **87**, 1044.

Ridgway S.T., Jacoby, G.H., Joyce, R.R., and Wells, D.C. 1980a, *Astron. J.*, **85**, 1496.

Ridgway, S.T., Joyce, R.R., White, N.M., and Wing, R.F. 1980b, *Astrophys. J.*, **235**, 126.

Ruland, F., Holweger, H., Griffin, R., and Biehl, D. 1980, *Astron. Astrophys.*, **92**, 70.

Saxner, M., and Gustafsson, B. 1984, *Astron. Astrophys.*, **140**, 334.

Scalo, J.M. 1973, *Astrophys. J.*, **186**, 967.

Scalo, J.M. 1976, *Astrophys. J.*, **206**, 474.

Scalo, J.M., Dominy, J.F., and Pumphrey, W.A. 1978, *Astrophys. J.*, **221**, 616.

Scalo, J.M., and Miller, G.E. 1979, *Astrophys. J.*, **233**, 596.

Scargle, J.D., and Strecker, D.W. 1979, *Astrophys. J.*, **228**, 838.

Schmid-Burgk, J., and Scholz, M. 1975, *Astron. Astrophys.*, **41**, 41.

Schmid-Burgk, J., and Scholz, M. 1981, *Mon. Not. Roy. Astr. Soc.*, **194**, 805.

Schmid-Burgk, J., Scholz, M., and Wehrse, R. 1981, *Mon. Not. Roy. Astr. Soc.*, **194**, 383.

Schmitz, F., and Ulmschneider, P. 1981, *Astron. Astrophys.*, **93**, 178.

Scholz, M., and Tsuji, T. 1984, *Astron. Astrophys.*, **130**, 11.

Scholz, M., and Wehrse, R. 1982, *Mon. Not. Roy. Astr. Soc.*, **200**, 41.

Schwartz, H.E., and Clarke, D. 1984, *Astron. Astrophys.*, **132**, 370.

Schwarzschild, M. 1975, *Astrophys. J.*, **195**, 137.

Sikorski, J. 1982, *Acta. Astr.*, **32**, 225.

Smith, M.A., and Dominy, J.F. 1979, *Astrophys. J.*, **231**, 477.

Sneden, C., Johnson, H.R., and Krupp, B.M. 1976, *Astrophys. J.*, **204**, 281.

Spiegel, E.A. 1963, *Astrophys. J.*, **138**, 216.

Steiman-Cameron, T.Y., and Johnson, H.R. 1985, *Astrophys. J.* (in press).

Steiman-Cameron, T.Y., Johnson, H.R., and Honeycutt, R.K. 1985, *Astrophys. J. (Letters)*, **291**, L51.

Stencel, R.E., and Mullan, D.J. 1980, *Astrophys. J.*, **238**, 221.

Strecker, D.W., Erickson, E.F., and Witteborn, F.C. 1979, *Astrophys. J. Supplement*, **41**, 501.

Strom, S.E., and Kurucz, R. 1966, *Jour. Quant. Spectr. and Rad. Transf.*, **6**, 591.

Thomas, R.N. 1960, *Astrophys. J.*, **131**, 429.

Thomas, R.N. 1983, *Stellar Atmospheric Structural Patterns*, NASA SP-471.

Thompson, R.I. 1977, *Astrophys. J.*, **212**, 754.

Toomre, J., Zahn, J.-P., Latour, J., and Spiegel, E.A. 1976, *Astrophys. J.*, **207**, 545.

Travis, L.D., and Matsushima, S. 1973a, *Astrophys. J.*, **180**, 975.

Travis, L.D., and Matsushima, S. 1973b, *Astrophys. J.*, **182**, 189.

Trimble, V., and Bell, R.A., 1981, *Quart. J. Roy. Astr. Soc.*, **22**, 361.

Tsuji, T. 1971, *Pub. Astron. Soc. Japan*, **23**, 553.

Tsuji, T. 1976a, *Pub. Astron. Soc. Japan*, **28**, 543.

Tsuji, T. 1976b, *Pub. Astron. Soc. Japan*, **28**, 567.

Tsuji, T. 1978a, *Astron. Astrophys.*, **62**, 29.

Tsuji, T. 1978b, *Pub. Astron. Soc. Japan*, **30**, 435.

Tsuji, T. 1981a, *Astron. Astrophys.*, **99**, 48.

Tsuji, T. 1981b, *J. Astrophys. Astron.*, **2**, 95.

Tsuji, T. 1981c, *J. Astrophys. Astron.*, **2**, 253.

Tsuji, T. 1983, *Astron. Astrophys.*, **122**, 314.

Tsuji, T., 1984, *Astron. Astrophys.*, **134**, 24.

Ulrich, R.K. 1970a, *Astrophys. Space Sci.*, **7**, 71.

Ulrich, R.K. 1970b, *Astrophys. Space Sci.*, **7**, 183.

Ulrich, R.K. 1970c, *Astrophys. Space Sci.*, **9**, 80.

Ulrich, R.K. 1976, *Astrophys. J.*, **207**, 564.

Vardya, M.S. 1970, *Ann. Rev. Astron. Astrophys.*, **8**, 87.

Vernazza, J.E., Avrett, E.H., and Loeser, R. 1981, *Astrophys. J. Supplement*, **45**, 635.

Walker, A.R. 1979, *South Africa Astr. Obs. Circ.*, **1**, p. 112.

Walker, A.R. 1980, *Mon. Not. Roy. Astr. Soc.*, **190**, 543.

Walker, A.R., Wild, P.A.T., and Byrne, P.B. 1979, *Mon. Not. Roy. Astr. Soc.*, **189**, 455.

Wallerstein, G. 1973, *Ann. Rev. Astron. Astrophys.*, **11**, 115.

Watanabe, T., and Kodaira, K. 1978, *Pub. Astron. Soc. Japan*, **30**, 21.

Watanabe, T., and Kodaira, K. 1979, *Pub. Astron. Soc. Japan*, **31**, 61.

Wehrse, R. 1981, *Mon. Not. Roy. Astr. Soc.*, **195**, 553.

Weiler, E.J., and Oegerle, W.R. 1979, *Astrophys. J. Supplement*, **39**, 537.

Wilson, O.C., and Bappu, M.K.V. 1957, *Astrophys. J.*, **125**, 661.

Wing, R.F. 1981, in *Physical Processes in Red Giants*, ed. I. Iben and A Renzini (Dordrecht: Reidel), p. 41.

Woodrow, J.E.J., and Auman, J.R. 1982, *Astrophys. J.*, **257**, 247.

Yorka, S.B. 1981, Ph.D. Thesis, Ohio State University.

Zuckerman, B. 1980, *Ann. Rev. Astron. Astrophys.*, **18**, 263.